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Olivier Cardi, Romain Restout

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Olivier CARDI
Romain RESTOUT

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DEPARTEMENT D'ECONOMIE
Route de Saclay
91128 PALAISEAU CEDEX
(33) 1 69333033
http://www.enseignement.polytechnique.fr/economie/
mailto:chantal.poujouly@polytechnique.edu

SECTORAL EFFECTS OF TAX REFORMS IN AN OPEN ECONOMY*

Olivier CARDI
Université Panthéon-Assas ERMES
Ecole Polytechnique

Romain RESTOUT
Université catholique de Louvain IRES

Abstract

We use a neoclassical open economy model with traded and non traded goods to investigate the sectoral effects of three tax reforms: i) two revenue-neutral shifting the tax burden from labor to consumption taxes and ii) one labor tax restructuring keeping the marginal tax wedge constant. Regardless of its type, a tax reform crowds-in both consumption and investment and raises employment. Whereas tax reforms have a small impact on GDP, they exert substantial effects on sectoral outputs which move in opposite direction in the short-run. The sensitivity analysis reveals that raising the elasticity of labor supply or reducing the tradable content in consumption expenditure amplifies the heterogeneity in sectoral output responses. Finally, allowing for the markup to depend on the number of competitors, we find that a substantial share of sectoral output variations can be attributed to the change in the markup triggered by firm entry.

Keywords: Non Traded Goods; Employment; Current Account; Tax Reform.
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1 Introduction

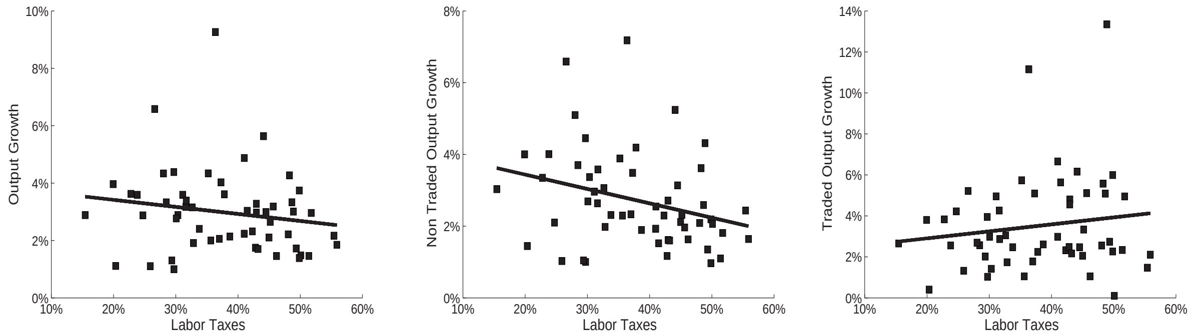
Tax reform is an important item on the current policy agenda and raises an academic enthusiasm. Most of the literature concludes that eliminating capital or labor income tax yields substantial beneficial effects. Using a neoclassical framework with liquidity-constrained consumers or human capital accumulation, Judd and Hubbard [1986] and Lucas [1990] have found positive effects on consumption, capital accumulation and GDP. Others have explored the effects of tax reforms in an open economy and reach similar conclusions, see e.g. Mendoza and Tesar [1998] and Coenen et al. [2008] who use two-country models of the neoclassical and of the new Keynesian variety, respectively. In particular, Mendoza and Tesar [1998] have shown that trade in world financial markets magnifies the welfare gains. Most of the analyses have been confined to one-sector models, however. Whereas so far conclusions have been drawn only for the aggregate economy, in the present paper, we take up the following question instead: what are the sectoral effects of a tax reform?

To estimate the sectoral effects of a tax reform, we consider an open economy with a traded and a non traded sector. Our neoclassical framework builds on Turnovsky and Sen [1995] and Coto-Martinez and Dixon [2003]. As Coto-Martinez and Dixon, we let the non traded sector to be imperfectly competitive. Our work differs from analyses by Turnovsky and Sen [1995] and Coto-Martinez and Dixon [2003] in one major respect. They investigate analytically the effects of government spending shocks whereas we provide both an analytical and a quantitative exploration of the effects of tax reforms. One attractive feature of a two-sector model with tradables and non tradables is to cover both the closed-economy and open-economy dimensions of contemporaneous industrialized countries. In particular, empirical evidence documents a sizeable non tradable share in GDP and total employment, averaging to 60% approximately.[1] A second key feature of a two-sector model is that a tax reform now produces a change in the relative price of non tradables which triggers a reallocation of resources between the two sectors. Third, our model allows to test if the labor intensive sector always benefits more from the labor tax cut. Fourth, such a model enables us to connect sectoral output responses to the trade balance adjustment.

To illustrate the potential importance in evaluating the effects of a tax reform at a sectoral level, we plot in the scatter diagrams of Figure 1 both GDP and sectoral output growth rates in percentage against the labor tax wedge for 27 OECD countries over the period 1994-2004 which has been split into two sub-periods 1994-1998 and 1999-2004.[2] Figure 1(b) suggests a negative

[1]Non tradable shares are reported in Appendix A.

[2]From the early Nineties, European tax systems were requested to achieve conflicting targets: reducing unemployment rate and achieving budget balance over the medium run. Hence, the period 1994-2004 is of



(a) Aggregate Economy (b) Non Traded Sector (c) Traded Sector

Figure 1: Growth and Labor Tax (1994-2004)

relationship between labor tax wedge and non traded output.[3] By contrast, as illustrated in Figure 1(c), traded output growth and labor tax wedge seem to show an opposite pattern. Furthermore, Figure 1(c) suggests a negative but small relationship between the GDP growth rate and labor taxes across countries. The model's predictions are in line with these findings: whereas traded and non traded sector vary in opposite direction in the short-run, a cut labor taxes exerts a small impact on GDP (see Figure 1(a)).

Since tax reforms take various forms, we consider three simple and practicable tax restructuring: i) two revenue-neutral tax reforms that reduce the marginal tax wedge by shifting the tax burden from labor to consumption taxes and ii) one labor tax reform keeping the marginal tax wedge constant by shifting the tax burden from employers to employees.[4] We show formally that regardless of its type, a tax restructuring crowds-in both consumption and investment, and raises employment. These results confirm earlier conclusions reached by Mendoza and Tesar [1998] who, in particular, experiment a tax reform replacing the labor income tax with a consumption tax within a two-country framework. The intuition behind these results are as follows. The fall in labor cost induces firms to raise wages which in turn stimulate labor supply. The consecutive increase in labor income pushes up consumption. To meet greater demand, the economy must accumulate capital. These conclusions are in line with the VAR

particular interest as a lot of countries compensate for labor tax cut by an increase in the consumption tax rate.

[3]Sample includes 27 countries: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Spain, Sweden, Switzerland, UK, US. The labor tax wedge is taken from OECD, Historical rates. Tax wedge includes income tax paid by workers and social security contributions levied on employees and their employers for a single person at 100% of average earnings, no child. Method of calculation of non traded and traded output is described in Appendix A. Source: KLEMS.

[4]While the two revenue-neutral tax reforms lower the marginal tax wedge, we consider a third strategy which involves simultaneously cutting payroll taxes and raising labor income taxes so as to keep the tax wedge constant. Whereas this labor tax restructuring does no longer keep the tax revenue fixed, it allows us to focus on the composition on the tax wedge rather than its level.

evidence documented by Blanchard and Perotti [2002] and Mertens and Ravn [2010], as long as the tax reform is unanticipated. Furthermore, like Mendoza and Tesar, we find that trade in world financial markets allows the open country to finance higher investment without sacrificing consumption in the short-run. Hence, the open economy runs short-run current account deficits which are compensated by a trade balance surplus in the long-run.

By differentiating between tradables and non tradables, our model yields new predictions at a sectoral level. Following a tax reform, the non traded sector strongly expands on impact while traded output falls dramatically. The reason is that the open country runs a deficit in the trade balance in the short-run triggered by the investment boom. The fall in net exports requires a drop in traded output which is achieved through a reallocation of resources towards the non traded sector. Henceforth, sectoral outputs move in opposite direction in the short-run, which implies that tax rates produce a small impact on GDP. By contrast, in the long-run, a tax reform stimulates both traded and non traded output. The reason is that the debt accumulated during the transition is serviced by a rise in net exports in the long-run. As consumption increases, such a surplus in the trade balance is achieved through a rise in traded output along the transitional path, triggered by the reallocation of resources towards the traded sector.

As it is currently assumed in the two-sector literature, for analytical simplicity, we first consider that the traded sector is more capital intensive than the non traded sector in discussing the macroeconomic effects of tax reforms.[5] Considering the case of reversal capital intensities, numerical experiments show that the effects are roughly similar. By contrast, we find that sectoral output responses are significantly sensitive to the elasticity of labor supply and the tradable content of consumption expenditure. First, irrespective of sectoral capital intensities, as labor supply gets more responsive, non traded output expands more while traded output falls by a larger amount on impact. Second, we find that a fall in the tradable content of consumption expenditure amplifies the heterogeneity in sectoral output responses, only when the non traded sector is relatively more capital intensive.

We also conduct a sensitivity analysis with respect to the degree of competition in the product markets, by making the markup endogenous.[6] The change in the markup provides an additional channel through which a tax reform impinges on sectoral outputs. We find that, by lowering the markup on impact, a tax reform stimulates further capital accumulation and yields a larger current account deficit. Hence, non traded output expands more while traded

[5] See e.g. Obstfeld [1989], Mendoza [1995], or Coto-Martinez and Dixon [2003] who assume that the traded sector is more capital intensive. Yet, our estimation of sectoral capital income shares in output show that the non traded sector is relatively more capital intensive in five countries over thirteen.

[6] Coto-Martinez and Dixon [2003] consider the case of a fixed markup.

output falls by a larger amount. The change of the markup amplifies the opposite responses of sectoral outputs, the net overall effect on GDP remaining roughly similar.

The remainder of the paper is organized as follows. Section 2 outlines the specification of a two-sector model with traded and non traded goods. In Section 3, we discuss the short-run and long-run effects of three tax reforms which involve cutting labor taxes. Section 4 provides a quantitative exploration of the sectoral effects and conducts a sensitivity analysis with respect to key parameters. In Section 5, we analyze to which extent our results are modified by considering that the non traded sector is more capital intensive. Section 6 explores quantitatively the case of an endogenous markup. Section 7 summarizes our main results and concludes.

2 The Framework

We consider a small open economy that is populated by a constant number of identical households and firms that have perfect foresight and live forever. The country is small in both world goods and capital markets and faces given world interest rate, $r^\star$. A perfectly competitive sector produces a traded good denoted by the superscript T that can be exported and consumed domestically. An imperfectly competitive sector produces a non traded good denoted by the superscript N which is devoted to physical capital accumulation and domestic consumption.[7] The traded good is chosen as numeraire.[8][9]

2.1 Households

At each instant the representative agent consumes traded goods and non traded goods denoted respectively by c^T and c^N, which are aggregated by a constant elasticity of substitution function:

$$c\left(c^T, c^N\right) = \left[\varphi^{\frac{1}{\phi}} \left(c^T\right)^{\frac{\phi-1}{\phi}} + (1-\varphi)^{\frac{1}{\phi}} \left(c^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{1}$$

with φ the weight attached to the traded good in the overall consumption bundle $(0 < \varphi < 1)$ and ϕ the intratemporal elasticity of substitution $(\phi > 0)$.

The agent is endowed with a unit of time and supplies a fraction $L(t)$ as labor and the remainder, $l \equiv 1 - L$ is consumed as leisure. At any instant of time, households derive utility

[7] As stressed by Turnovsky and Sen [1995], allowing for traded capital investment would not affect the results. Furthermore, like Burstein et al. [2004], non tradable investment accounts for a large share of total investment (i.e. 60%).

[8] The price of the traded good is determined on the world good market and exogenously given for the small open economy.

[9] More details on the model as well as the derivations of the results which are stated below are provided in an Appendix which is available on request.

from their consumption and leisure. Households decide on consumption and worked hours by maximizing lifetime utility:

$$U = \int_0^\infty \left\{ \frac{1}{1-\frac{1}{\sigma_c}} c(t)^{1-\frac{1}{\sigma_c}} - \gamma \frac{1}{1+\frac{1}{\sigma_L}} L(t)^{1+\frac{1}{\sigma_L}} \right\} e^{-\beta t} \mathrm{d}t, \tag{2}$$

where β is the consumer's discount rate, $\sigma_c > 0$ is the intertemporal elasticity of substitution for consumption, $\sigma_L > 0$ is the Frisch elasticity of labor supply;

Factor income is derived from supplying labor L at a wage rate w and capital K at a rental rate r^K.[10] Labor is taxed at rate τ^H. The wage tax is levied on households' wage income above a certain threshold κ, which represents the personal tax allowance. Thus, $w^A = w - (w - \kappa)\,\tau^H$ corresponds to the after-tax wage. As long as tax allowances are positive, the tax system is progressive which means that the average tax burden rises with the wage rate. In addition, households accumulate internationally traded bonds, $b(t)$, that yields net interest rate earnings $r^\star b(t)$. Denoting by Z lump-sum transfers from the government, the households' flow budget constraint writes as follows:

$$\dot{b}(t) = r^\star b(t) + r^K(t)K(t) + w^A(t)L(t) + Z - p_c\,(p(t))\,(1+\tau^c)\,c(t) - p(t)I(t), \tag{3}$$

where p_c is the consumption price index which is a function of the relative price of non traded goods p. The last two terms represent households' expenditure which include purchases in consumption goods inclusive of consumption tax rate τ^c, and investment expenditure pI. Aggregate investment gives rise to overall capital accumulation according to the following dynamic equation:

$$\dot{K}(t) = I(t) - \delta_K K(t). \tag{4}$$

where we assume that physical capital depreciates at rate δ_K. From now thereon, the time-argument is suppressed for clarity purpose.

Denoting by λ the co-state variable associated with equation (3), the first-order conditions characterizing the representative household's optimal plans are:

$$c = \left(p_c\,(1+\tau^c)\,\lambda\right)^{\sigma_c}, \tag{5a}$$

$$L = \left(\frac{\bar{\lambda}}{\gamma_L} w^A\right)^{\sigma_L}, \tag{5b}$$

$$\dot{\lambda} = \lambda\,(\beta - r^\star), \tag{5c}$$

$$\frac{r^K}{p} - \delta_K + \frac{\dot{p}}{p} = r^\star, \tag{5d}$$

and the appropriate transversality condition. To generate an interior solution for the marginal utility of wealth λ, we require the time preference rate to be equal to the world interest rate.

[10]We abstract from capital income tax which is beyond the scope of this paper.

This standard assumption made in the literature implies that the marginal utility of wealth, λ, must remain constant over time, i. e. $\lambda = \bar{\lambda}$.

2.2 Firms

There are two sectors of production in the model producing a traded good T and a non traded good N. The traded and non traded sectors face two cost components: a capital rental cost equal to r^K, and a labor cost equal to $w^F = w\left(1+\tau^F\right)$ with τ^F the the employer's part of labor taxes.

The traded sector is assumed to be perfectly competitive and use capital K^T and labor L^T according to a constant returns to scale production function, $Y^T = F\left(K^T, L^T\right)$, which has the usual neoclassical properties of positive and diminishing marginal products. The first order conditions derived from profit-maximization in the traded sector state that factors are paid to their respective marginal products.

The final non traded output, Y^N, is produced in a competitive retail sector using a constant-returns-to-scale production which aggregates a continuum measure one of sectoral non traded goods.[11] We denote by $\omega > 0$ the elasticity of substitution between any two different sectoral goods. In each sector, there are $N > 1$ firms producing differentiated goods that are aggregated into a sectoral non traded good. We denote by $\epsilon > 0$ the elasticity of substitution between any two varieties. We assume that the elasticity of substitution between any two goods within a sector is higher than the elasticity of substitution across sectors, i.e. $\epsilon > \omega$ (see e.g. Jaimovich and Floetotto [2008]). Within each sector, there is monopolistic competition; each firm that produces one variety is a price setter. Output $\mathcal{X}_{i,j}$ of firm i in sector j is produced using capital and labor, i.e. $\mathcal{X}_{i,j} = H\left(\mathcal{K}_{i,j}, \mathcal{L}_{i,j}\right)$. Each firm chooses capital and labor by equalizing markup-adjusted marginal products to the marginal cost of inputs, i. e. $H_K/\mu = r^K$, and $H_L/\mu = w^F$, where μ is the markup over marginal costs. Aggregate non traded output is equal to $Y^N = N\mathcal{X} = H\left(K^N, L^N\right)$. We assume that there is a large number of firms within each sector so that each single intermediate producer is small relative to the economy and thereby each producer in one sector faces a constant price elasticity of demand ϵ. Hence, one producer of a variety charges a constant markup denoted by $\mu = \frac{e}{e-1}$ with e the price-elasticity of demand equal to ϵ as the number of competitors is large. In section 6, we relax this assumption and rather assume that a finite number of firms operate within each sectors producing non tradable varieties.[12] We further assume instantaneous entry which implies that the zero profit condition

[11] The setup builds on Jaimovich and Floetotto [2008] and thereby details of derivation are relegated to the Appendix available on request.

[12] As stressed by Yang and Heijdra [1993], departing from the usual assumption made by Dixit and Stiglitz [1977] implies that the price elasticity of demand becomes an increasing function of the number of firms and

holds at each instant of time.

Denoting by $k^i \equiv K^i/L^i$ the capital-labor ratio for sector $i = T, N$, we express the production functions in intensive form, that is $f\left(k^T\right) \equiv F\left(K^T, L^T\right)/L^T$ and $h\left(k^N\right) \equiv H\left(K^N, L^N\right)/L^N$. Production functions are supposed to take a Cobb-Douglas form: $f\left(k^T\right) = \left(k^T\right)^{\theta^T}$, and $h\left(k^N\right) = \left(k^N\right)^{\theta^N}$, where θ^T and θ^N represent the capital income share in output in the traded and non traded sector, respectively. Since inputs can freely move between the two sectors, marginal products in the traded and the non traded sector equalize:

$$\theta^T \left(k^T\right)^{\theta^T-1} = \frac{p}{\mu}\theta^N \left(k^N\right)^{\theta^N-1} \equiv r^K, \tag{6a}$$

$$\left(1-\theta^T\right)\left(k^T\right)^{\theta^T} = \frac{p}{\mu}\left(1-\theta^N\right)\left(k^N\right)^{\theta^N} \equiv w^F. \tag{6b}$$

Static efficiency conditions (6a)-(6b) state that sectoral marginal revenue products must equalize to the labor producer cost w^F and capital rental rate r^K. System (6a)-(6b) can be solved for sector capital intensities ratios: $k^T = k^T(p)$ and $k^N = k^N(p)$.

Aggregating labor and capital over the two sectors, we obtain the resource constraints for both inputs:

$$L^T + L^N = L, \quad K^T + K^N = K. \tag{7}$$

2.3 Government

The final agent in the economy is the government who finances lump-sum transfers to households Z together with public spending falling on the traded g^T and the non traded good pg^N by raising taxes on consumption, $\tau^c p_c c$, and labor, $\left[\tau^H(w-\kappa)+\tau^F w\right]L$, according to the following balanced condition:

$$\tau^c p_c c + \left[\tau^H(w-\kappa)+\tau^F w\right]L = Z + g^T + pg^N. \tag{8}$$

2.4 Macroeconomic Dynamics

The adjustment of the open economy towards the steady-state is described by a dynamic system which comprises the dynamic equation for the relative price of non traded goods (5d) which equalizes the return on domestic capital and traded bonds $r^\star$. The second equation is the accumulation equation for physical capital which clears the non traded good market along the transitional path. Solving first-order conditions for output and consumption, the market clearing condition for the non traded good writes as:

$$\dot{K} = Y^N(K, L, p)/\mu - c^N\left(\bar{\lambda}, p, \tau^c\right) - g^N - \delta_K K, \tag{9}$$

that the markup turns out to be endogenous.

where $L = L\left(\bar{\lambda}, p, \tau^F, \tau^H, \mu\right)$ is the short-run static solution of eq. (5b); consumption in the non traded good c^N is equal to $\alpha_c p_c c/p$ with α_c is the non tradable content in consumption expenditure. Solving eq. (5a) for consumption, the short-run static solution for c^N follows. It is worthwhile to notice that a labor tax cut, i.e. a fall in τ^F or τ^H, affects the macroeconomic equilibrium by modifying labor supply decisions.

Dynamic equations (5d)-(9) form a separate subsystem in p and K.[13] Denoting by a tilde the steady-state value, stable solutions for K and p write as:

$$K(t) - \tilde{K} = \left(K_0 - \tilde{K}\right) e^{\nu_1 t}, \quad p(t) - \tilde{p} = \omega_2^1 \left(K(t) - \tilde{K}\right), \tag{10}$$

where $\omega_2^1 \leq 0$ is the eigenvector associated with stable eigenvalue ν_1. If $k^T > k^N$, we have $\omega_2^1 = 0$, so that the relative p, consumption, labor, and thereby savings adjust immediately to their steady-state levels. By contrast, with the reversal of capital intensities, transitional dynamics for the consumption-side variables are restored as $\omega_2^1 < 0$. Substituting (9) and (8) into (3), we obtain the dynamic equation for the current account denoted by ca:

$$\dot{b} = r^\star b + Y^T(K, L, p) - c^T\left(\bar{\lambda}, p, \tau^c\right) - g^T, \tag{11}$$

where $c^T = (1 - \alpha_c) p_c c$ is consumption in the traded good with $(1 - \alpha_c)$ the tradable content in consumption expenditure. Equation (11) states that the current account is equal to the trade balance denoted by nx, i.e. $nx \equiv Y^T - c^T - g^T$, plus interest receipts on outstanding assets.

2.5 Steady-State

We now discuss the salient features of the steady-state. Setting $\dot{p} = 0$ into eq. (5d), we obtain the equality between the after-tax rate of return on domestic capital income $\theta^N \left(\tilde{k}^N\right)^{\theta^N - 1}/\mu - \delta_K$ and the exogenous world interest rate, $r^\star$, that determines the steady-state value of the relative price of non tradables $\tilde{p}$. The long-run level of p remains unaffected by a tax restructuring, as long as the markup is fixed. The steady-state level of p determines the wage rate $\tilde{w} = \frac{\theta^T \left[k^T(\tilde{p})\right]^{\theta^T - 1}}{1+\tau^F}$. By substituting the wage rate into the labor supply decision evaluated at the steady-state, we get $\tilde{L} = \left\{\frac{\bar{\lambda}}{\gamma_L}\left[\tilde{w} - (\tilde{w} - \kappa)\tau^H\right]\right\}^{\sigma_L}$. For given $\bar{\lambda}$, a cut in τ^F raises the wage rate and thereby stimulates labor supply.

Setting $\dot{K} = 0$ into eq. (9) yields the market-clearing condition for the non traded good:

$$\frac{1}{\mu} Y^N\left(\tilde{K}, \tilde{L}, \tilde{p}\right) = c^N\left(\bar{\lambda}, \tilde{p}, \tau^c\right) + \tilde{I} + g^N, \tag{12}$$

[13] Since the number of predetermined variables (K) equals the number of negative eigenvalues (denoted by ν_1) and the number of jump variables (p) equals the number of positive eigenvalues (denoted by ν_2), the equilibrium yields a unique one-dimensional stable saddle-path, irrespective of the relative sizes of sectoral capital-labor ratios.

where $\tilde{I} = \delta_K \tilde{K}$.

Setting $\dot{b} = 0$ into eq. (11) yields the market-clearing condition for the traded good:

$$Y^T \left(\tilde{K}, \tilde{L}, \tilde{p}\right) = -r^\star \tilde{b} + c^T \left(\bar{\lambda}, \tilde{p}, \tau^c\right) + g^T. \tag{13}$$

The intertemporal solvency condition can be solved for the shadow value of wealth:

$$\left(\tilde{b} - b_0\right) = \Omega \left(\tilde{K} - K_0\right), \tag{14}$$

where $\Omega < 0$ describes the effect of capital accumulation on the the external asset position and K_0 and b_0 are the initial stocks of capital and foreign assets.[14]

Eqs. (12)-(14) jointly determine the steady-state values of physical capital, $\tilde{K}$, foreign bonds holding, $\tilde{b}$, and the shadow value of wealth, $\bar{\lambda}$. It is worthwhile to note that eq. (14) connects eqs. (12) and (13), and thereby sectoral outputs. More precisely, an investment boom $\tilde{I}$ stimulates non traded output $\tilde{Y}^N$ and traded output $\tilde{Y}^T$ too since capital accumulation yields a drop in $\tilde{b}$. The reason is that the fall in interest receipts due to lower traded bonds holding must be exactly matched by a long-run improvement in the balance of trade which exerts a positive impact on $\tilde{Y}^T$.

3 Effects of Tax Reforms: An Analytical Exploration

Since tax reforms can take various forms, we consider three types of tax restructuring. We explore two revenue-neutral tax reforms which involve simultaneously either cutting payroll taxes by $\mathrm{d}\tau^F < 0$ or labor income taxes by $\mathrm{d}\tau^H < 0$ and raising the consumption tax by $\mathrm{d}\tau^c > 0$. While these tax reforms cause a fall in the tax wedge, we consider a third tax restructuring which involves simultaneously cutting payroll taxes by $\mathrm{d}\tau^F < 0$ and raising the wage income tax rate $\mathrm{d}\tau^H > 0$ that leaves unchanged the tax wedge. The third policy allows us to analyze a shift in the composition of the labor taxation rather than a change in the marginal tax wedge defined as the difference between the producer wage and the after-tax marginal wage expressed as a percentage of the producer cost: $\tau^M = 1 - \frac{(1-\tau^H)}{(1+\tau^F)}$ (see e.g. Heijdra and Ligthart [2009]). Moreover, For pedagogical purpose, we assume that the traded sector is more capital intensive than the non traded sector, i.e. $k^T > k^N$, as it is commonly assumed in the two-sector literature. We explore the case of reversal capital intensities in section 5.

[14] If $k^T > k^N$, then $\Omega = -\tilde{p} < 0$. If $k^N > k^T$, $\Omega = -\tilde{p} \left[1 + \frac{\omega_2^1}{\tilde{p}\nu_2} \left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L} \tilde{k}^T \left(\nu_2 + \delta_K\right) \tilde{\Lambda}\right)\right]$ with $0 < \Lambda \equiv \frac{(1-\tau^H)}{\left[(1-\tau^H) + \frac{\tau^H \kappa}{w}\right]} < 1$.

3.1 Revenue-Neutral Tax Reforms

We analyze first the long-run effects of a revenue-neutral tax reform. To avoid confusion, we denote by $\big|^{j,c}$ the effects of a fall in the labor tax by $\mathrm{d}\tau^j < 0$ $(j = F, H)$ coordinated with a rise in the consumption tax rate by $\mathrm{d}\tau^c\big|^{j,c}$ which is endogenously determined so as the government budget constraint is met. Assuming that the stock of financial wealth is positive, the labor tax base is smaller than the consumption tax base.[15] Hence, τ^c must increase less than the drop in labor tax.[16]

The long-term change of $x = c, L, K, nx$ following a shift from labor tax to consumption tax is equal to the sum of the expansionary impact of the labor tax cut by $\mathrm{d}\tau^j < 0$ $(j = F, H)$ financed by lump-sum taxes (i.e. $\frac{\partial \tilde{x}}{\partial \tau^j}\mathrm{d}\tau^j > 0$) and the recessionary effect triggered by the rise in the consumption tax rate by $\mathrm{d}\tau^c\big|^{j,c}$ (i.e. $\frac{\partial \tilde{x}}{\partial \tau^c}\mathrm{d}\tau^c\big|^{j,c} < 0$):[17]

$$\mathrm{d}\tilde{x}\big|^{j,c} = \frac{\partial \tilde{x}}{\partial \tau^j}\mathrm{d}\tau^j + \frac{\partial \tilde{x}}{\partial \tau^c}\mathrm{d}\tau^c\big|^{j,c} \equiv \Phi^{j,c}\frac{\partial \tilde{x}}{\partial \tau^j}\mathrm{d}\tau^j > 0, \quad j = F, H, \tag{15}$$

where $0 < \Phi^{j,c} < 1$ $(j = F, H)$. The second equality of eq. (15) states that the long-run change in $x = c, L, K, nx$ following a tax reform is simply a scaled-down version of the long-term changes of x after a lump-sum tax financing labor tax cut. Consequently, a tax reform shifting the labor tax to the consumption tax stimulates employment and consumption, and raises both the capital stock and net exports. The reason is that a labor tax cut induces agents to supply more labor. The consecutive increase in the after-tax labor income boosts consumption. In the same time, the labor inflow in the non traded sector raises output in that sector which in turn stimulates capital accumulation. The capital inflow in the traded sector raises its output which results in a long-run improvement in the balance of trade. A tax reform produces the same effects but now their size are moderated due to the scaling-down term $0 < \Phi^{j,c} < 1$.

Moreover, a tax reform stimulates further c, L, K, nx as the scaled-down term $\Phi^{j,c}$ gets closer to unity. If the labor tax cut was financed by a rise in lump-sum taxes, $\Phi^{j,c}$ would be equal to unity, and the higher bound of the net overall outcome of a tax reform would be obtained. In contrast, if the stock of financial wealth was equal to zero, the consumption tax

[15] At the steady-state, denoting by $a \equiv b + pK$ the stock of financial wealth, we have: $r^\star \tilde{a} + Z + \tilde{w}^A \tilde{L} = p_c(1 + \tau^c)\tilde{c}$. As long as $r^\star \tilde{a} + Z > 0$, the consumption tax base is larger than the labor tax base.

[16] Both the labor tax cut and the rise in τ^c yields opposite effects on tax receipts. On the one hand, a labor tax cut lowers public revenue, keeping unchanged consumption and employment. Second, a labor tax rate cut raises employment and consumption, and thereby tax revenues. While analytically, the net overall effect cannot be signed, we find numerically that the former effect always more than offsets the latter effect. The same logic applies to a change in the consumption tax rate. We find numerically that the net overall effect on tax revenues of a labor tax cut is close to that following a rise in the consumption tax, in absolute terms. Hence, only the ratio of the labor tax base to the consumption tax base matters in determining the size of the increase in τ^c.

[17] Formal details can be retrieved in the Appendix.

base would be equal to the labor tax base, and a tax reform would produce no effect.[18] This situation corresponds to the lower bound of the net overall outcome of a tax reform. The effects of a tax restructuring falls between these two bounds.[19]

Let now discuss the impact effects. In the case $k^T > k^N$, the dynamics of the relative price degenerate so that labor and real consumption increase immediately to their final long-term levels. Investment is the result of demand and supply reactions in the non traded good market. The increase in total employment induces a labor inflow in the non traded sector which boosts Y^N. While the initial rise in c^N withdraws resources from capital accumulation, the stimulus of non traded output is large enough to cause an investment boom on impact. Formally, using stable solution (10), we have: $\mathrm{d}I(0)\big|^{j,c} = -\nu_1 \mathrm{d}\tilde{K}\big|^{j,c} > 0$. Finally, the open country runs a trade balance deficit in the short run given by $\mathrm{d}nx(0)\big|^{j,c} = \nu_1 \tilde{p} \mathrm{d}\tilde{K}\big|^{j,c} < 0$, reflecting the immediate boom in investment as savings remain unchanged.

So far, we have analyzed the effects of a labor tax cut coordinated with a rise in the consumption tax rate, without differentiating between a cut in the employer's (i.e. τ^F) or employee's (i.e. τ^H) part of labor taxes. A drop in τ^H leaves unaffected w and raises the after-tax labor income by $(\tilde{w} - \kappa)$. A cut in τ^F raises the wage rate and thereby the after-tax labor income by $\tilde{w}\frac{1-\tau^H}{1+\tau^F} = \tilde{w}\left(1 - \tau^M\right)$, with τ^M the marginal tax wedge. Whereas the size of the effects after a fall in τ^H decreases with tax progressiveness, the magnitude of the effects following a fall in τ^F rises with the marginal tax wedge.

3.2 A Labor Tax Restructuring

As it is common in the literature investigating the macroeconomic effects of a tax reform, so far we have analyzed revenue-neutral tax reforms. Let now consider that the policy maker wishes to alter the composition of the marginal tax wedge without however, changing its level. We denote by the superscript $\{F, H\}$ the effects of a tax reform which involves simultaneously cutting the employer's part of labor tax and increasing the personal income tax $\mathrm{d}\tau^H > 0$ so as to leave unchanged the marginal tax wedge (i. e. $\mathrm{d}\tau^M = 0$). A labor tax restructuring requires a rise in the personal income tax by an amount given by:

$$\mathrm{d}\tau^H\big|^{F,H} \equiv -\frac{1-\tau^H}{1+\tau^F}\mathrm{d}\tau^F = \left(\tau^M - 1\right)\mathrm{d}\tau^F > 0. \tag{16}$$

According to (16), the personal income tax must be increased by a smaller amount than the fall in τ^F for keeping unchanged the marginal tax wedge. Intuitively, since the tax rate on a

[18] More rigorously, the labor tax base is equal to the consumption tax base if $r^\star \tilde{a} + Z = 0$.

[19] The larger the share of financial wealth in real disposable income, the greater the consumption tax base compared to the labor tax base, and thereby the closer to one the scaling-down term $\Phi^{j,c}$. Hence, the less τ^c needs to increase for a given labor tax cut to balance the budget and the larger the effects of a tax reform on c, L, K, nx.

relatively large base is reduced and the tax rate on a relatively small base is increased, the latter must rise by a smaller proportion than the former decreases so as to leave unchanged τ^M.

The steady-state change of $x = c, L, K, nx$ following a cut in τ^F, coordinated with a rise in τ^H by an amount given by (16), reads:

$$\mathrm{d}\tilde{x}\Big|^{F,H} = \Phi^{F,H}\frac{\partial \tilde{x}}{\partial \tau^F}\mathrm{d}\tau^F = \frac{\kappa}{\tilde{w}}\mathrm{d}\tau^F, \tag{17}$$

where $0 < \Phi^{F,H} \equiv \kappa/\tilde{w} < 1$. Setting κ to zero implies that such a tax reform will produce no effects on the economy. Rather, as long as the labor tax scheme is progressive, i. e. $\kappa > 0$, the labor tax reform leaving constant the tax wedge raises permanently x. As for revenue-neutral tax reforms, the steady-state changes in c, L, K, nx are a scaled-down version of their long-term changes following a labor tax cut financed by lump-sum taxes. The scaled-down term is equal to $\kappa/\tilde{w}$ and thereby depends on the degree of progressiveness of the tax scheme. The stronger the progressiveness in the tax scheme, the larger the increase in the after-tax wage rate and thereby the greater the beneficial effects on employment and overall economic activity. We dot not discuss further the impact effects which are similar to that described for revenue-neutral strategies.

3.3 Output Response

We now investigate in details the response of output at an overall level and importantly at a sectoral level. We denote by $\Big|^{j,k}$ the effects of a fall in the labor tax by $\mathrm{d}\tau^j < 0$ ($j = F, H$) financed by a rise in τ^k ($k = c, H$).

3.3.1 GDP Response

We analyze first the response of GDP to highlight the role of trade balance. Using the fact that in the long-run, overall output equalizes its demand counterpart, and differentiating, the long-run GDP response is given by:[20]

$$\mathrm{d}\tilde{Y}\Big|^{j,k} = p_c\mathrm{d}\tilde{c}\Big|^{j,k} + \mathrm{d}\tilde{nx}\Big|^{j,k} + \tilde{p}\mathrm{d}\tilde{I}\Big|^{j,k} > 0, \tag{18}$$

where $\mathrm{d}\tilde{x}\Big|^{j,k} > 0$ (with $\tilde{x} = \tilde{c}, \tilde{I}, \tilde{nx}$) is given by (15) or (17) depending on the type of the tax reform. According to (18), the domestic demand boom for both consumption and investment goods stimulates GDP. Furthermore, the improvement in the balance of trade to service the

[20]Using the market-clearing condition, i. e. $\frac{\tilde{Y}^N}{\mu} = \tilde{c}^N + g^N + \tilde{I}$, and the current account, i. e. $\tilde{Y}^T = (\tilde{c}^T + g^T) - r^\star\tilde{b}$, aggregating and differentiating, we get (18), keeping in mind that the steady-state level of the relative price of non tradables remains unchanged.

debt accumulated during the transition raises further $\tilde{Y}$. In a closed economy framework, the latter demand component vanishes. Hence, as stressed by Mendoza and Tesar [1998], a closed economy model would underestimate the beneficial effects of a tax restructuring since the trade balance surplus magnifies the effects of a tax reform on GDP in the long-run.

Linearizing aggregate demand for the domestic good, i. e. $Y = p_c(p(t))c(t) + \left(g^T + p(t)g^N\right) + p(t)I(t) + nx(t)$, evaluating at time $t = 0$ and differentiating enables us to decompose the GDP response in its demand counterparts as follows:[21]

$$\mathrm{d}Y(0)\big|^{j,k} = p_c \mathrm{d}c(0)\big|^{j,k} + \tilde{p}\mathrm{d}I(0)\big|^{j,k} + \tilde{Y}^N \mathrm{d}p(0)\big|^{j,k} + \mathrm{d}nx(0)\big|^{j,k} > 0, \tag{19}$$

where $\mathrm{d}p(0)\big|^{j,k} = 0$ as long as $k^T > k^N$ since the dynamics for the relative price degenerate in this case. According to (19), the initial response of GDP is driven by the initial demand boom for both consumption and investment goods. Yet, as reflected by the last term, the trade balance deficit lowers the size of the GDP increase. As long as $k^T > k^N$, savings remain unchanged. Hence, the worsening in the external asset position mirrors exactly the investment boom. Consequently, the initial response of GDP is only driven by higher consumption. Since consumption adjusts immediately to its long-run level, the short-run increase in GDP is smaller than that in the long-run.[22]

3.3.2 Sectoral Output Responses

We now analyze if the traded and non traded sectors are affected uniformly by a tax reform. Using the market-clearing condition together with the zero current account equation, we can derive the steady-state changes of non traded and traded output, respectively:

$$\frac{1}{\mu}\mathrm{d}\tilde{Y}^N\big|^{j,k} = \mathrm{d}\tilde{c}^N\big|^{j,k} + \mathrm{d}\tilde{I}\big|^{j,k} > 0, \tag{20a}$$

$$\mathrm{d}\tilde{Y}^T\big|^{j,k} = \mathrm{d}\tilde{nx}\big|^{j,k} + \mathrm{d}\tilde{c}^T\big|^{j,k} > 0. \tag{20b}$$

Since a tax restructuring raises after-tax labor income and induces households to consume more, demands for both traded and non traded consumption goods expand. Additionally, higher investment in physical capital and net exports raise further non traded and traded output, respectively. Interestingly, in the long-run, sectoral outputs are positively correlated. More precisely, the larger the economic boom in the non traded sector, the more traded output increases in the long-run. The explanation is that the greater the investment boom is, the larger the accumulated debt and the more net exports must increase.

[21] An alternative way to determine the initial response of GDP is to use the fact that $Y \equiv Y^T + \frac{p}{\mu}Y^N$. Keeping in mind that the capital stock is initially predetermined and differentiating, we obtain $\mathrm{d}Y(0)\big|^{j,k} = w^F \mathrm{d}L(0)\big|^{j,k} > 0$. Since worked hours increase on impact, GDP rises in the short-run.

[22] In the case $k^N > k^T$, we reach similar conclusions since savings play little role as we consider time separable preferences and we assume that the tax reform is permanent.

We now evaluate if the sectoral outputs move in opposite direction. The sectoral output responses in the short-run are:

$$\frac{1}{\mu}\mathrm{d}Y^N(0)\big|^{j,k} = \mathrm{d}c^N(0)\big|^{j,k} + \mathrm{d}I(0)\big|^{j,k} > 0, \tag{21a}$$

$$\mathrm{d}Y^T(0)\big|^{j,k} = \mathrm{d}nx(0)\big|^{j,k} + \mathrm{d}c^T(0)\big|^{j,k} \gtrless 0. \tag{21b}$$

According to (21a), the demand boom for non tradables causes an expansion in the non traded sector. With regard to the traded sector, the dramatic drop in net exports on impact now counteracts the positive influence of higher consumption. If $k^T > k^N$, it can be proven analytically that traded output falls on impact. The intuitive explanation is that households get richer due to a higher after-tax wage and greater labor supply. Hence, they are induced to consume further. They raise c^N and c^T. But since the traded sector is more capital intensive, it experiences a labor outflow on impact so that traded output declines. Hence, the rise in c^T reflects additional imports which results in a trade balance deficit.[23]

4 Tax Reforms: A Quantitative Exploration

In this section, we analyze the effects of tax reforms quantitatively. For this purpose we solve the model numerically. In the following, we thus first discuss parameter values before turning to the long-term and short-term effects of the tax substitutions.

4.1 Benchmark Parametrization

We start by describing the calibration of consumption-side parameters that we use as a baseline. The world interest rate, which is constrained to equalize the subjective time discount rate β, is chosen to be 3%. The intertemporal elasticity of substitution σ_c is set to 0.7 and the intratemporal elasticity of substitution ϕ to 2 (see e.g. Cashin and Mc Dermott [2003]). One critical parameter is the intertemporal elasticity of substitution for labor supply σ_L. In our baseline parametrization, we set $\sigma_L = 0.5$, in line with evidence reported by Domeij and Flodén [2006]. An additional critical parameter is φ which is set to 0.5 in the baseline calibration to target a tradable content in total consumption expenditure (i.e., $1-\alpha_c$) of 50%. Below, we conduct a sensitivity analysis with respect to these two parameters, i.e. we set σ_L to 0.2 and 1, and φ to 0.1 and 0.9.[24] For reason of space, we focus on the shift of employers' labor taxes (i.e. a fall in τ^F) towards consumption tax (i.e. a rise in τ^c) in evaluating the sensitivity of numerical results to σ_L and φ.

[23] More precisely, physical capital accumulation requires a shift of resources towards the non traded sector. Hence, traded output falls. Because consumption in traded good rises, additional demand yields higher imports.

[24] Raising φ from 0.1 to 0.9 increases the tradable share in GDP Y^T/Y from 24% to 62%.

We now describe the calibration of production-side parameters. We let physical capital to depreciate at a rate $\delta_K = 4\%$ to generate an investment-GDP ratio of 20% which is consistent with data from developed countries. Sectoral capital income shares in output take two different values depending on whether the traded sector is more or less capital intensive than the non traded sector. In line with our estimates, when $k^T > k^N$, the values of θ^T and θ^N are set to 0.4 and 0.3 respectively. Alternatively, in the case $k^N > k^T$, we choose $\theta^T = 0.3$ and $\theta^N = 0.4$.[25] The elasticity of substitution between varieties of non traded goods, ϵ is set to 3 to target a markup of 1.5, in line with our estimates (see Appendix A). In the case of an endogenous markup explored in section 6, keeping ϵ unchanged, we set the elasticity os substitution between sectoral goods ω to 2 to target a markup of 1.5.

To set τ^c, τ^F and τ^H, we estimated effective tax rates for thirteen OECD countries over 2000-2007. The consumption tax τ^c is set to 13%, the employer's part of labor taxes τ^F to 16% and the wage income tax τ^H to 31%. In evaluating quantitatively the effects of a tax reform, we consider a labor tax cut by 5 percentage points. Tax allowances κ is set to 0.3 to obtain a share of taxable income into the gross wage earnings $(w - \kappa)/w$ of 0.7. We set g^N and g^T to target a non tradable share of government spending of 90% and government spending as a share of GDP of 20%.

4.2 Long-Run Effects: A Quantitative Sectoral Decomposition

We now provide a quantitative exploration of the size of long-run effects.

4.2.1 Macroeconomic Effects

Using simple algebra, we have shown previously that long-term changes of consumption, employment and capital stock following a tax restructuring are a scaled-down version of the steady-state changes following a lump-sum tax financing labor tax cut. For the baseline parametrization, we find that the scaling-down term displays the same magnitude across the types of tax reforms. More precisely, for our benchmark parametrization, $\Phi^{j,k}$ is equal to 0.25, approximately. Hence, the size of the long-run effects are similar across the three tax reforms.[26]

As discussed in section 3, a tax restructuring stimulates both consumption and investment, and improves the balance of trade. To disentangle the contribution to each GDP component

[25] $\theta^T = 0.4$ and $\theta^N = 0.3$ correspond approximately to average for countries with $k^T > k^N$ (see Table 3). In the case of reversal capital intensities, we consider symmetric values for θ^N and θ^T so that the gap between sectoral capital intensities remain unchanged.

[26] For our baseline calibration, as the tax wedge is high and the tax scheme displays weak progressiveness of tax progressiveness, a tax reform which involves cutting the wage income tax paid by households τ^H while raising the consumption tax rate produces the larger effects on c, L, K.

to the rise in overall output, we scaled steady-state changes of consumption, investment and the balance of trade by initial GDP. Numerical results are summarized in Table 1B. We find that two-third of the GDP growth is driven by the rise in consumption. The remaining share is attributed to the steady-state improvement in the balance of trade and the investment boom.

4.2.2 GDP Response

Table 1D summarizes the numerical values for both overall and sectoral output responses. For the benchmark parametrization, the steady-state response of GDP falls in the range between 0.26-0.30, depending on the tax reform which is implemented. More importantly, as emphasized previously, sectoral outputs are positively correlated in the long-run. Interestingly, we find that that tradable output expands more than non tradable output: $\tilde{Y}^T$ rises by 0.15% of GDP while $\tilde{Y}^N$ increases by 0.11%. The sizeable expansion in the traded sector relies upon the long-run improvement in the balance of trade. While the tradable share in overall output is about 40%, its contribution to GDP increase is close to 60%.

4.3 Impact Effects: A Quantitative Sectoral Decomposition

The sectoral decomposition of the effects of a tax restructuring allows to highlight the propagation mechanism. The impact responses of sectoral outputs are summarized in the second and third line of Table 1E. Interestingly, sectoral outputs vary in opposite direction in the short-run. More precisely, the increase in non traded output is four times larger than that of GDP and falls between 0.67% and 0.79% of initial GDP approximately, depending on the tax restructuring which is implemented. By contrast, the traded sector experiences a severe decline in its output which falls between 0.5% and 0.59% of initial GDP approximately. The reason for opposite responses in sectoral output stems from the shift of resources across sectors. The initial stimulus of hours worked shifts resources from the traded towards the non traded sector which is labor intensive. In the same time, households consume more due to higher after-tax labor income. While tradable output decreases, about half of the additional income is devoted to imports which causes a current account deficit. In section 5.1, we find that sectoral outputs also vary in opposite direction in the case of reversal capital intensities, i.e. $k^N > k^T$.

4.4 Dynamics Effects

We now investigate the dynamic effects. Computed transitional paths of key variables are displayed in Figures 2 where we consider a shift of the tax burden from labor taxes (i. e. a cut in τ^F) to consumption taxes (i. e. a rise in τ^c). Investment and the current account are scaled by initial GDP while sectoral outputs are expressed as deviations from initial steady-state values scaled by initial GDP (in percentage). Reactions of sectoral employment are scaled by initial total employment.

As illustrated in Figures 2(b)-2(c), employment increases by about 0.7% in the non traded sector which boosts Y^N by 0.67%. Conversely, the traded sector experiences a labor outflow by almost -0.5% which drives down its output by 0.5%. As shown in Figure 2(a), the labor inflow in the non traded sector triggers an investment boom which drives the current account into deficit in the short-run by -0.5% of GDP. As the economy accumulates physical capital over the transition, the traded sector which is more capital intensive expands. As displayed in Figures 2(b)-2(c), the investment boom shifts resources towards the traded sector so that employment and thereby output rises in that sector. After 3 years, traded output exceeds its initial level and converges towards a new higher steady-state level. While the initial boom in the non traded sector slows down, the rate of growth remains positive over the entire adjustment.

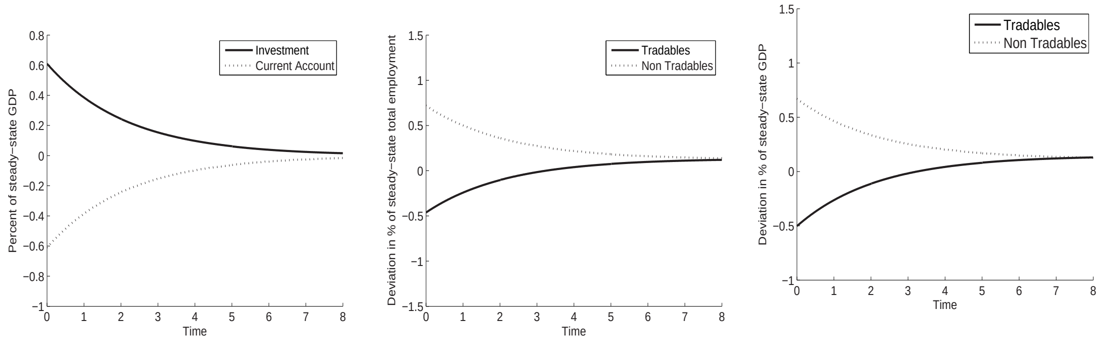
4.5 Sensitivity Analysis

As expected, the size of labor supply responsiveness exerts a sizeable effect on steady-state GDP response and GDP components. As σ_L is raised from 0.2 and 1, the GDP response increases from 0.08% to 0.48% approximately. By contrast, raising the tradable share of consumption by increasing φ does not modify significantly the long-run GDP response. The reason is that the relative price remains unchanged in the long-run.

Interestingly, labor supply also affects significantly the contribution of traded output to the long-run GDP expansion. While setting σ_L to 0.2 implies that half of the GDP growth can be attributed to the traded sector, its contribution is close to 60% if σ_L is set to 1. The reason is that, as labor supply gets more responsive, the traded sector experiences a greater labor outflow on impact which triggers a larger current account deficit. Hence, net exports must increase more in the long-run which boosts further $\tilde{Y}^T$.

Additionally, increasing the tradable good content (i.e. raising φ) plays a significant role in driving the size of the long-term responses of sectoral outputs. Setting $\varphi = 0.9$ implies that the growth in traded output contributes to 80% of GDP increase. The reason is that consumption in the traded good expands by a larger amount which magnifies the increase in $\tilde{Y}^T$. Since GDP growth is unaffected by the value of φ, a rise in the traded good content shifts resources towards the traded good sector.

Finally, raising σ_L amplifies considerably the heterogeneity in sectoral output responses in the short-run. Increasing σ_L from 0.2 to 1, the decline in Y^T becomes more pronounced (-0.90% against -0.16%) and the rise in Y^N gets larger (1.20% against 0.21%). Because the relative price remains unchanged, raising φ does not modify significantly the magnitude of



(a) Investment and Current Account (b) Sectoral Employment (c) Sectoral Output

Figure 2: Transitional Paths after a Labor-Consumption Tax Restructuring (Fixed Markup)

sectoral output responses.

5 A more Capital Intensive Non Traded Sector

So far, we have assumed that the traded sector was more capital intensive than the non traded sector, as it is commonly assumed in the two-sector theoretical literature for analytical convenience. Yet, our estimation of sectoral shares of capital income in output show that half of industrialized countries displays a non traded sector which is more capital intensive. Given its empirical relevance and for robustness purpose, we analyze to which extent our results are modified by considering that the non traded sector is more capital intensive. Such an analysis allows us to test if the non traded sector still benefits the labor tax cut in the short-run, even if it is more capital intensive. Furthermore, we evaluate the sensitivity of our numerical results to the tradable share in consumption expenditure and labor supply responsiveness.

5.1 The Role of Sectoral Capital Intensities

Assuming that $k^N > k^T$ implies that the dynamics for p do no longer degenerate. Rather, the relative price p must rise on impact as the result of higher demand for non tradables and decline along the transitional path to equalize the returns on domestic and foreign assets. Since the temporal path of the relative price is no longer flat, transitional dynamics for consumption and labor are restored. Importantly, the non traded sector experiences a labor outflow which shifts towards the more labor intensive sector. Because the relative price of non tradables strongly appreciates on impact, the consecutive shift of resources triggers an investment boom. With regard to the long-term GDP response, Table 1D shows that the results are weakly sensitive to sectoral capital intensities.

Table 1: Quantitative Effects of Tax Reforms (Exogenous Markup)

	$k^T > k^N$							$k^N > k^T$						
	Benchmark			Sensitivity analysis				Benchmark			Sensitivity analysis			
	Rev-Neut		Const. τ^M	Labor Supply		Trad. Share		Rev-Neut		Const. τ^M	Labor Supply		Trad. Share	
	$d\tau^F$	$d\tau^H$	$d\tau^F$	$\sigma_L = 0.2$	$\sigma_L = 1$	$\varphi = 0.1$	$\varphi = 0.9$	$d\tau^F$	$d\tau^H$	$d\tau^F$	$\sigma_L = 0.2$	$\sigma_L = 1$	$\varphi = 0.1$	$\varphi = 0.9$
A.Tax Rate changes														
$\mathrm{d}\tilde{\tau}^c$	+3.4	+4.0		+3.9	+3.1	+3.4	+3.4	+3.2	+3.7		+3.6	+3.0	+3.2	+3.3
$\mathrm{d}\tilde{\tau}^F$	-5.0		-5.0	-5.0	-5.0	-5.0	-5.0	-5.0		-5.0	-5.0	-5.0	-5.0	-5.0
$\mathrm{d}\tilde{\tau}^H$		-5.0	+3.0						-5.0	+3.0				
B.Steady-State Effects														
Labor, $\mathrm{d}\tilde{L}$	0.26	0.30	0.26	0.08	0.46	0.27	0.25	0.29	0.33	0.29	0.12	0.48	0.30	0.29
Real Wage, $\mathrm{d}\tilde{w}$	4.52	0.00	4.52	4.52	4.52	4.52	4.52	4.52	0.00	4.52	4.52	4.42	4.52	4.52
Output, $\mathrm{d}\tilde{Y}$	0.26	0.30	0.27	0.08	0.47	0.27	0.25	0.29	0.33	0.29	0.12	0.47	0.30	0.28
Consumption, $\mathrm{d}\tilde{c}$	0.17	0.20	0.17	0.05	0.30	0.18	0.16	0.19	0.21	0.19	0.08	0.30	0.18	0.19
Investment, $\mathrm{d}\tilde{I}$	0.06	0.05	0.06	0.02	0.09	0.05	0.05	0.05	0.07	0.05	0.02	0.09	0.06	0.05
Net exports, $\mathrm{d}\tilde{nx}$	0.04	0.05	0.04	0.01	0.08	0.04	0.04	0.05	0.05	0.05	0.02	0.08	0.06	0.04
C.Impact Effects														
Labor, $\mathrm{d}L(0)/\tilde{L}_0$	0.26	0.30	0.26	0.08	0.46	0.27	0.25	0.23	0.26	0.23	0.11	0.28	0.23	0.23
Real Wage, $\mathrm{d}w(0)/\tilde{w}_0$	4.52	0.00	4.52	4.52	4.52	4.52	4.52	4.37	-0.17	4.37	4.46	4.30	4.36	4.38
Output, $\mathrm{d}Y(0)/\tilde{Y}_0$	0.17	0.20	0.17	0.05	0.30	0.18	0.16	0.15	0.17	0.15	0.07	0.18	0.14	0.15
Consumption, $\mathrm{d}c(0)/\tilde{Y}_0$	0.17	0.20	0.17	0.05	0.30	0.18	0.16	0.18	0.20	0.18	0.07	0.29	0.17	0.19
Investment, $\mathrm{d}I(0)/\tilde{Y}_0$	0.61	0.71	0.63	0.19	1.09	0.57	0.63	0.61	0.69	0.61	0.25	0.99	0.67	0.55
Net Exports, $\mathrm{d}nx(0)/\tilde{Y}_0$	-0.61	-0.71	-0.63	-0.19	-1.09	-0.57	-0.63	-0.64	-0.72	-0.64	-0.25	-1.10	-0.70	-0.59
D.Sect. Decomp. Long-Run														
$\mathrm{d}\tilde{Y}$	0.26	0.30	0.27	0.08	0.47	0.27	0.25	0.29	0.33	0.29	0.12	0.47	0.30	0.28
$\mathrm{d}\tilde{Y}^T$	0.15	0.17	0.15	0.04	0.27	0.07	0.19	0.15	0.17	0.15	0.06	0.24	0.07	0.22
$\mathrm{d}\tilde{Y}^N$	0.11	0.13	0.12	0.04	0.20	0.20	0.06	0.14	0.16	0.14	0.06	0.23	0.23	0.06
E.Sect. Decomp. Impact														
$\mathrm{d}Y(0)$	0.17	0.20	0.17	0.05	0.30	0.18	0.16	0.15	0.17	0.15	0.07	0.18	0.14	0.15
$\mathrm{d}Y^T(0)$	-0.50	-0.59	-0.52	-0.16	-0.90	-0.54	-0.48	-0.53	-0.60	-0.53	-0.20	-0.92	-0.67	-0.41
$\mathrm{d}Y^N(0)$	0.67	0.79	0.69	0.21	1.20	0.72	0.64	0.68	0.77	0.68	0.27	1.10	0.81	0.56

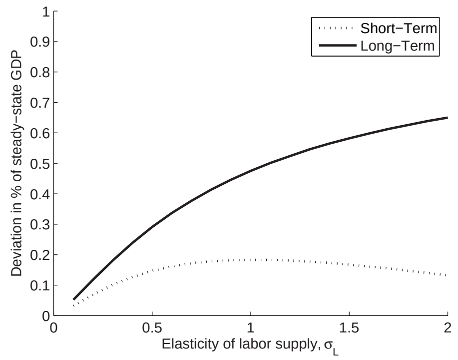
5.2 Tradable Share

In contrast to the case $k^T > k^N$, short-run sectoral output responses are sensitive to the tradable content in consumption expenditure. More precisely, raising φ moderates significantly the heterogeneity of output responses across sectors. As summarized in the second line of Table 1E, traded output falls by about -0.67% on impact if the tradable content is low (i.e. φ is set to 0.1) whereas its decline is less severe if φ is set to 0.9. The reason is that as the share of tradables in consumption expenditure increases, the excess of demand in the non traded good market gets smaller. Consequently, the relative price of non tradables appreciates by a lower amount which in turn moderates both the decline in Y^T and the rise in Y^N. Yet, the short-run response of GDP remains unaffected by φ.

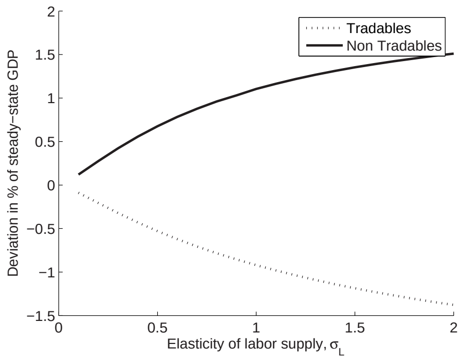
5.3 Labor Supply responsiveness

As in the case $k^T > k^N$, the elasticity of labor supply σ_L plays a major role in determining the size of both GDP and sectoral output responses in the long-run. Increasing σ_L from 0.2 to 1 raises the long-run GDP response from 0.12% to 0.47% approximately, as shown in Table 1D. To have further insight, we conduct a sensitivity analysis with respect to σ_L which is allowed to vary from 0.1 to 2. Figure 3(a) plots both the long-run and initial response of GDP against σ_L while Figure 3(b) plots both traded and non traded output impact responses against the sensitivity of labor supply.

As illustrated in Figure 3(a), like Baxter et King [1993] who use a one-sector model, the GDP response in the long-run rises with σ_L. As labor gets more sensitive to the rise in the after-tax labor income, the non traded sector experiences a larger labor outflow. The consecutive greater excess of demand for non tradables requires a larger increase in the stock of capital to clear the non traded good market. Yet, unlike Baxter and King [1993], the relationship between the short-run response of GDP and the elasticity of labor supply displays a hump-shaped pattern. This non monotonic relationship can be best understood by using a sectoral decomposition. Interestingly, Figure 3(b) shows that, as labor supply gets more responsive, non traded output rises more whereas traded output declines further. The explanation is as follows. As labor supply is more responsive, the excess of demand in the non traded good market gets larger, so that the relative price of non tradables appreciates more on impact. The larger increase in p raises further Y^N but lowers Y^T more. As long as $\sigma_L > 1$, raising labor supply responsiveness implies that the larger decline in traded output more than offsets the stronger increase in non traded output which results in a smaller rise in GDP, as shown in Figure 3(a).



(a) GDP response against σ_L



(b) Short-run sectoral output response against σ_L

Figure 3: Sensitivity of Output Responses to Labor Supply Responsiveness

6 Endogenous Markups and Sectoral Effects of Tax Reforms

Recently, several papers have stressed that the variation in the number of competitors and the consecutive change in the markup provides an important magnification mechanism, see e.g. Jaimovich and Floetotto [2008], Wu and Zhang [2000], Zhang [2007] who consider one-sector models. We thus investigate the sectoral effects of tax reforms by allowing for the markup to be endogenous.[27]

6.1 Steady-state Effects

So far, we adopted the Dixit-Stiglitz assumption according to which the number of competitors is large enough within each sector to yield a fixed price-elasticity of demand. Yet, as emphasized by Yang and Heijdra [1993], the assumption of Dixit and Stiglitz [1977] is an approximation when the final good is aggregated by a finite number of intermediate goods. We depart from the usual practice, following Galí [1994], in assuming that the number of firms is large enough so that we can ignore the strategic effects but not so large that the effect of entry is minuscule on the firm's demand curve. Consequently, the price elasticity of demand faced by a single firm is no longer constant and equal to the elasticity of substitution between any two varieties, but rather a function of the number of firms N. Taking into account that output of one variety does not affect the general price index p, but influences the sectoral price level, in a symmetric equilibrium, the resulting price elasticity of demand writes as:[28]

$$e\left(N\right) = \epsilon - \frac{(\epsilon - \omega)}{N}, \quad N \in (1, \infty). \tag{22}$$

Assuming that $\epsilon > \omega$, the price elasticity of demand faced by one single firm is an increasing function of the number of firms N within a sector. Henceforth, the markup $\mu = \frac{e}{e-1}$ decreases

[27]We assume that the traded sector is more capital intensive than the non traded sector for clarity purpose. Numerical results for the case $k^N > k^T$ are available upon request.

[28]Details of derivation can be found in the Appendix.

as the number of competitors increases.

In the interest of space, we restrict attention to the major changes in deriving the macroeconomic equilibrium. First, the zero-profit condition in the intermediate good sector can be solved for the number of firms, i.e. $N = N(K, L, p)$. Keeping in mind that $\mu = \mu(N)$, equalities of marginal products between sectors (6) imply that capital-labor ratios k^j $(j = T, N)$ are affected by the markup and thereby the number of firms, i.e. $k^j = k^j(p, \mu)$. Substituting the capital-labor ratios into the sectoral marginal products of labor and the resource constraints, we obtain the short-run static solutions for the wage rate and sectoral outputs:

$$w = w\left(p, \tau^F, \mu\right), \quad Y^T = Y^T(K, p, L, \mu), \quad Y^N = Y^N(K, p, L, \mu). \tag{23}$$

The wage rate and sectoral output are now affected by a *competition effect* triggered by the change in the markup. A larger number of firms N lowers the markup μ which increases the wage rate (since we assume that $k^T > k^T$). Because households raise labor supply, non traded output Y^N increases while traded output Y^T falls.

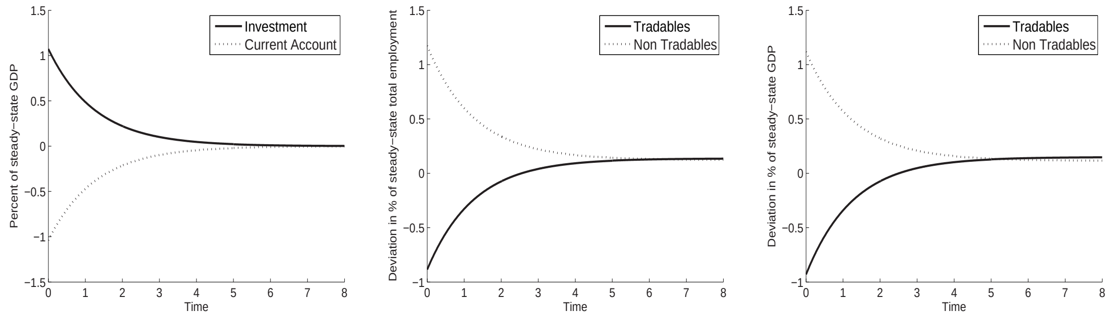
Let now discuss how the change of the markup affects the steady-state effects of a tax reform. First, the markup depends on the number of firms which adjusts to drive profits to zero, i.e. $\tilde{Y}^N\left(1 - \frac{1}{\mu(\tilde{N})}\right) = \tilde{N}\chi$ where χ represents the fixed costs. In the light of our discussion in section 3.3, a tax reform raises non traded output. Hence, average cost falls which thereby fosters firms' entry. A higher number of firms reduces the markup. A lower μ leads to a long-run fall in the relative price of non tradables p, regardless of sectoral capital intensities, to equalize the rates of return on domestic and foreign assets, i.e. $\theta^N\left(\tilde{k}^N\right)^{\theta^N-1}/\mu\left(\tilde{N}\right) - \delta_K = r^\star$. Inspection of eqs. (23) which hold at the steady-state, reveals that changes in the relative price and in the markup impinge on sectoral outputs. The decline in the relative price shifts resources away from the non traded towards the traded sector. Yet, the *competition effect* reflected by a fall in the markup counteracts the *relative price effect*. Numerically, we find that these two effects offset so that long-run sectoral effects remain similar to those found in the case of fixed markup.[29]

6.2 Impact Effects: A Sectoral Decomposition

We now investigate the short-term effects in the case of an endogenous markup. Interestingly, while the *competition channel* does not modify the long-run GDP response as the *relative price effect* works in opposite direction, the fall in the markup exerts sizeable effects in the short-run, in particular on sectoral outputs.

As the non traded output overshoots its steady-state level on impact which lowers significantly average cost, the number of firms initially shoots up before slowly converging towards

[29]Numerical results for long-term effects of a tax reform in the case of an endogenous markup are available on request.



(a) Investment and Current Account (b) Sectoral Employment (c) Sectoral Output

Figure 4: Transitional Paths after a Labor-Consumption Tax Restructuring (Endogenous Markup)

its new higher steady-state value. The consecutive drop in μ is large enough to foster further capital accumulation, though the initial decline in the relative price of non tradables works in opposite direction of the competition channel. As reported in Table 2A, the investment boom is almost two times larger than in the case of fixed markup, i.e. rises from 0.6% to 1.1% of initial GDP. Additionally, individuals are more willing to supply labor because the substantial drop in the markup raises the wage rate. While the open economy experiences a larger investment boom, the GDP response is similar to that in the case of fixed markup since the balance of trade enters in a greater deficit.

Table 2: Quantitative Effects of Tax Reforms (Endogenous Markup - $k^T > k^N$)

	Benchmark			Sensitivity analysis	
	Rev-Neut		Const. τ^M	Labor Supply	
	$d\tau^F$	$d\tau^H$	$d\tau^F$	$\sigma_L = 0.2$	$\sigma_L = 1$
A.Impact Effects					
Labor, d$L(0)$	0.29	0.34	0.29	0.08	0.55
Real Wage, d$w(0)$	4.40	-0.15	4.40	4.47	4.33
Output, d$Y(0)$	0.19	0.22	0.19	0.05	0.36
Consumption, d$c(0)$	0.13	0.16	0.14	0.05	0.31
Investment, d$I(0)$	1.10	1.29	1.13	0.37	1.97
Net Exports, d$nx(0)$	-1.04	-1.21	-1.06	-0.35	-1.84
B.Sect. Decomp. Impact					
d$Y(0)$	0.19	0.22	0.19	0.05	0.36
d$Y^T(0)$	-0.93	-1.08	-0.95	-0.31	-1.65
d$Y^N(0)$	1.12	1.30	0.14	0.36	2.01

Interestingly, as summarized in Table 2B, the fall in the markup magnifies considerably the heterogeneity in sectoral output responses on impact. As discussed previously, considering an endogenous markup into the analysis amplifies substantially the investment boom. Moreover, the fall in p induces agents to consume more of the non traded good. By stimulating further the demand for non tradables, a tax restructuring raises non traded output by 1.1% instead of 0.7% in the case of fixed markup. In the same time, the greater deficit in the balance of trade results in a larger decline in traded output. As illustrated in Figure 4(c), the drop in the markup amplifies the sectoral output responses.

Over time, firms' entry slows down. The relative price of non tradables appreciates to equalize the rate of return between domestic and foreign assets. The increase in p along the transitional path lowers c^N and raises c^T over time. As depicted in Figure 4(b), capital accumulation shifts employment away from the non traded sector towards the traded sector, as in the case of fixed markup. As time passes, the gap between sectoral output growth shrinks, as illustrated in Figure 4(c). The intersectoral reallocations are strong enough to raise the traded output growth above the non traded output growth in the long-run. The same explanation developed in the case of fixed markup applies. The open economy finances the investment boom along the transitional path by a current account deficit which must be matched by an improvement in the trade balance in the long-run. Such a rise in net exports is achieved though the increase in Y^T originating from a labor inflow after about 3 years.

7 Conclusion

We used a two-sector small open economy producing both traded and non traded goods to investigate the short-run and long-run effects of three tax restructuring. We consider two budget-neutral strategies that shifts the payroll or personal labor income taxes to consumption taxes and one strategy keeping the marginal tax wedge constant that reduces the taxes paid by employers and raises the taxes paid by employees. Our conclusions confirm earlier findings reached by Mendoza and Tesar [1998]: cutting the labor income tax and raising the consumption tax, leaving unchanged the government budget, crowds-in both consumption and investment, and raises employment and GDP.

Our paper also complements earlier studies by investigating the sectoral effects of tax reforms. We find that traded and non traded outputs are negatively correlated in the short-run but are positively correlated in the long-run. The reason is that the open economy finances capital investment without lowering consumption by running a current account deficit. While the investment boom raises sharply non traded output in the short-run, the fall in net exports triggers a significant decline in traded output on impact. As sectoral outputs move in opposite direction over the transition, a tax reform has a small impact on GDP. In the long-run, the open economy runs a trade balance surplus to service the debt accumulated over the transition. As resources shifts towards the traded sector, output in that sector increases. Interestingly, for the baseline calibration, roughly 60% of GDP growth originates from the expansion in traded output.

Furthermore, the sensitivity analysis shows that raising the elasticity of labor supply significantly amplifies responses of sectoral outputs both in the short-run and the long-run, regardless of sectoral capital intensities. While traded output falls by a larger amount on impact as labor supply responsiveness rises, output in the traded sector increases more in the long-run. We also find that raising the tradable content in consumption expenditure lowers substantially the magnitude of sectoral output responses, as long as the non traded sector is more capital intensive, since in this case the relative price of non tradables increases less.

Building on Jaimovich and Floetotto [2008], we endogenize the markup charged by the non traded sector. Numerical results reveal that the short-run fall in the markup amplifies considerably sectoral output responses by stimulating further investment and triggering a larger current account deficit. Yet, the response of GDP remains almost unaffected.

A Data

We split the overall economy into a traded and non traded sector. Table 3 reports the non tradable share of GDP, employment, consumption expenditure, and gross fixed capital formation for 13 OECD countries. The choice of these countries has been dictated by data availability. We follow the methodology proposed by De Gregorio et al. [1994], who treat Agriculture, Hunting, Forestry and Fishing, Mining and Quarrying, Total Manufacturing, Transport and Storage and Communication as traded goods. Electricity, Gas and Water Supply, Construction, Wholesale and Retail Trade, Hotels and Restaurants, Finance, Insurance, Real Estate and Business Services, Community Social and Personal Services are classified as non traded sectors.

With regard to investment, we follow the methodology proposed by Burstein et al. [2004] who treat Housing and Other Construction as non tradable investment and Products of agriculture, forestry, fisheries and aquaculture, Metal products and machinery, Transport Equipment as tradable investment expenditure (source: OECD Input-Output database).

For reason of space, we did not report the non tradable share of government spending which averages to 90% for the countries of our sample. Sectoral government expenditure data over the period 1978-2004 were obtained from the Government Finance Statistics Yearbook and OECD database. Following Morshed and Turnovsky [2004], the following four sectors were treated as traded: Fuel and Energy; Agriculture, Forestry, Fishing, and Hunting; Mining, Manufacturing, and Construction; Transport and Communications. The following sectors were treated as being non traded: Government Public Services; Defense; Public Order and Safety; Education; Health; Social Security and Welfare; Housing and Community Amenities; Recreation Cultural and Community Affairs.

Markups are estimated at the industry level for each country, classified as non traded sectors, and are aggregated as follows to construct μ:

$$\mu = \sum_{j=1}^{6} \omega_j \; \hat{\mu}_j, \tag{24}$$

where ω_j is the nominal value added-weight of industry j in the non traded sector. Estimates $\hat{\mu}_j$ are obtained by applying the consistent methodology developed by Roeger [1995]. Inputs are labor, capital and intermediate; variables required to apply the Roeger's method are the following: gross output (at basic current prices), compensation of employees, intermediate inputs at current purchasers prices, and capital services (volume) indices. The testable equation of the Roeger's methodology may be written as:

$$y_{j,t} = \beta_j \, x_{j,t} + \varepsilon_{j,t}, \tag{25}$$

with $y_t = \Delta \left(p_{j,t} Y_{j,t}\right) - \alpha_{N,t} \Delta \left(w_{j,t} L_{j,t}\right) - \alpha_{M,t} \Delta \left(m_{j,t} M_{j,t}\right) - \left(1 - \alpha_{N,t} - \alpha_{M,t}\right) \Delta \left(r_t K_{j,t}\right)$, $x_{j,t} = \Delta \left(p_{j,t} Y_{j,t}\right) - \Delta \left(r_t K_{j,t}\right)$, and $\varepsilon_{j,t}$ the i.i.d. error term. $\Delta \left(p_{j,t} Y_{j,t}\right)$ denotes the nominal output growth in industry j, $\Delta \left(w_{j,t} L_{j,t}\right)$ the nominal labor cost growth, $\Delta \left(m_{j,t} M_{j,t}\right)$ the growth in nominal intermediate input costs and $\Delta \left(r_t K_{j,t}\right)$ the nominal capital cost growth. All these variables are compiled from the EU KLEMS database, with the exception of the user cost of capital r_t. No sector-specific information was available to construct r_t; hence, the rental price of capital is calculated as $r_t (\equiv r_{j,t}) = p_I \left(i - \pi_{GDP} + \delta_K\right)$, with p_I the deflator for business non residential investment, i the long-term nominal interest rate, π_{GDP} the GDP deflator based inflation rate; the rate of depreciation δ is set to 5%; p_I, i and π_{GDP} were taken from OECD database. An econometric issue arises when estimating (232) with the OLS is the potential endogeneity of the regressor associated with the heteroskedasticity and autocorrelation of the error term. To tackle these problems, we estimated (232) by using heteroskedastic and autocorrelation consistent standard errors as suggested by Newey and West [1993] (lag truncation =2). Finally, the markup estimate $\hat{\mu}_j$ is equal to $1/(1 - \hat{\beta}_j)$.[30] Results are reported in Table 3.

To estimate the tax rates of consumption and labor, we use the OECD database. We split labor taxes into employee's and employer's part of labor taxes. Payroll tax, personal income tax, and consumption tax are effective tax rates and are computed according to the following formulas:

$$\tau^F = \frac{\text{Taxes on payroll and workforce} + \text{Employers' contribution to social security}}{\text{Compensation of employees}},$$

$$\tau^H = \text{Income tax (average rate)} + \text{Employees' social security contributions (average rate)},$$

$$\tau^c = \frac{\text{Taxes on production, sale, transfer}}{\text{Final consumption expenditure of households and general government}}.$$

[30] Countries estimates for each $\hat{\mu}_j$, $j = 1, ..., 11$, are not reported here to save space, but are available upon request.

Table 3: Data to Calibrate the Two-Sector Model

Countries	Non tradable Share				Capital Share		Markup	Taxes			
	Y^N/Y	L^N/L	c^N/c	I^N/I	θ^T	θ^N	μ	τ^c	τ^F	τ^H	$(w-\kappa)/w$
Austria	0.65	0.60	0.44	0.59	0.28	0.32	1.52	0.16	0.16	0.32	0.77
Belgium	0.67	0.65	0.44	n.d.	0.33	0.35	1.39	0.11	0.20	0.42	0.80
Denmark	0.70	0.67	0.43	0.58	0.32	0.32	1.52	0.21	0.00	0.42	0.88
Spain	0.61	0.59	0.44	0.63	0.35	0.26	1.37	0.12	0.22	0.20	0.66
Finland	0.58	0.57	0.40	0.63	0.27	0.30	1.41	0.19	0.23	0.32	0.93
France	0.69	0.64	0.44	0.61	0.22	0.35	1.42	0.14	0.27	0.29	0.60
Germany	0.64	0.61	0.36	0.54	0.22	0.33	1.55	0.13	0.15	0.44	0.92
Italy	0.63	0.56	0.39	0.59	0.42	0.39	1.73	0.13	0.27	0.28	0.82
Japan	0.64	0.61	0.45	0.63	0.37	0.29	1.63	0.06	0.08	0.19	0.50
Netherlands	0.67	0.69	0.50	0.64	0.41	0.33	1.36	0.15	0.09	0.33	0.96
Sweden	0.65	0.67	0.51	0.47	0.30	0.30	1.44	0.17	0.23	0.31	0.94
UK	0.62	0.66	0.52	0.52	0.30	0.28	1.47	0.13	0.07	0.26	0.83
US	0.68	0.72	0.49	0.59	0.36	0.32	1.42	0.05	0.06	0.25	0.78

Notes: Y^N/Y, L^N/L, c^N/c and I^N/I are the non tradable share in GDP, employment, consumption, and gross fixed capital formation; θ^j is the GDP share of capital income in sector $j = T, N$; μ is the markup charged in the non traded sector; τ^c is the consumption tax rate, τ^F the employers' part of labor taxes, and τ^H: the employees' part of labor taxes. Source: IMF [2007], OECD [2008a], [2008a], United Nations [2007] and EU KLEMS [2007].

Tax allowances, κ, are calculated as the share of taxable income into the gross wage earnings before taxes.

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SECTORAL EFFECTS OF TAX REFORMS IN AN OPEN ECONOMY

TECHNICAL APPENDIX

NOVEMBER 2010

Olivier CARDI and Romain RESTOUT

A Short-Run Static Solutions

A.1 Short-Run Static Solutions for Consumption-Side

Since $c(.)$ is homothetic, the household's maximization problem can be decomposed into two stages. In the first stage, households choose their real consumption, c, labor supply L, and the rates of accumulation of traded bonds b together with domestic capital K by maximizing (2) subject to the flow budget constraints (3) and (4), and initial conditions $b(0) = b_0$ and $K(0) = K_0$:[31]

$$u_c(c) = p_c(p)(1+\tau^c)\lambda, \tag{26a}$$

$$v_L(L) = -\lambda\left[w - (w-\kappa)\tau^H\right], \tag{26b}$$

$$\dot{\lambda} = \lambda(\beta - r^\star), \tag{26c}$$

$$\frac{r^K}{p} + \frac{\dot{p}}{p} = r^\star, \tag{26d}$$

where λ is the co-state variable associated with dynamic equation (3).

Static efficiency conditions (26a) and (26b) can be solved for real consumption and labor which of course must hold at any point of time:

$$c = c\left(\bar{\lambda}, p, \tau^c\right), \quad L = L\left(\bar{\lambda}, p, \tau^F, \tau^H, \mu\right), \tag{27}$$

with

$$c_{\bar{\lambda}} = \frac{\partial c}{\partial \bar{\lambda}} = -\sigma_c \frac{c}{\bar{\lambda}} < 0, \tag{28a}$$

$$c_p = \frac{\partial c}{\partial p} = -\alpha_c \sigma_c \frac{c}{p} < 0, \tag{28b}$$

$$c_{\tau^c} = \frac{\partial c}{\partial \tau^c} = -\sigma_c \frac{c}{(1+\tau^c)} < 0, \tag{28c}$$

$$L_{\bar{\lambda}} = \frac{\partial L}{\partial \bar{\lambda}} = \sigma_L \frac{L}{\bar{\lambda}} > 0, \tag{28d}$$

$$L_p = \frac{\partial L}{\partial p} = \sigma_L L \frac{w_p\left(1-\tau^H\right)}{w^A} = -\sigma_L L \frac{\Lambda}{w^F} \frac{k^T h}{\mu\left(k^N - k^T\right)} \lesseqgtr 0, \tag{28e}$$

$$L_{\tau^F} = \frac{\partial L}{\partial \tau^F} = -\sigma_L L \frac{w_{\tau^F}\left(1-\tau^H\right)}{w^A} = -\sigma_L L \frac{\Lambda}{\left(1+\tau^F\right)} < 0, \tag{28f}$$

$$L_{\tau^H} = \frac{\partial L}{\partial \tau^H} = -\sigma_L L \frac{(w-\kappa)}{w^A} < 0, \tag{28g}$$

$$L_\mu = \frac{\partial L}{\partial \mu} = \sigma_L L \frac{w_\mu\left(1-\tau^H\right)}{w^A} = \sigma_L L \frac{\Lambda}{w^F} \frac{k^T p h}{(\mu)^2\left(k^N - k^T\right)} \gtreqless 0, \tag{28h}$$

where $\sigma_c = -\frac{u_c}{u_{cc}c} > 0$ corresponds to the intertemporal elasticity of substitution for consumption, $\sigma_L = \frac{v_L}{v_{LL}L} > 0$ represents the intertemporal elasticity for labor. We denoted by

[31] The transversality conditions are:

$$\lim_{t\to\infty} p(t)K(t)e^{-r^\star t} = \lim_{t\to\infty} b(t)e^{-r^\star t} = 0.$$

$0 < \Lambda \equiv \frac{\left(1-\tau^H\right)}{\left[\left(1-\tau^H\right)+\frac{\tau^H \kappa}{w}\right]} < 1$ as longer as $\kappa > 0$; if $\kappa = 0$, then $\Lambda = 1$. According to (28f), labor supply decreases or increases following a real exchange rate appreciation depending on wether $k^N \gtrless k^T$.

According to (28), a rise in agent's wealth reflected by a fall in the shadow value of of wealth $\bar{\lambda}$ stimulates consumption while discouraging labor supply. By raising the cost of living, a rise in the relative price of non tradables p lowers c. The wage rate w and thereby labor supply is lowered or raised following an increase in p depending on whether $k^N \gtrless k^T$. While a rise in the consumption tax rate τ^c depresses consumption by raising its marginal cost, a fall in the labor income tax rate levied on employers τ^F or employees τ^H stimulates labor supply by raising the after-tax labor income.

Denoting by ϕ the intratemporal elasticity of substitution between the tradable and the non tradable good and inserting short-run solution for consumption (27) into intra-temporal allocations between non tradable and tradable goods, we solve for c^T and c^N:

$$c^T = c^T\left(\bar{\lambda}, p, \tau^c\right), \quad c^N = c^N\left(\bar{\lambda}, p, \tau^c\right), \tag{29}$$

with

$$\begin{aligned}
c^T_{\bar{\lambda}} &= -\sigma_c \frac{c^T}{\bar{\lambda}} < 0, & (30a)\\
c^T_p &= \alpha_c \frac{c^T}{p}\left(\phi - \sigma_c\right) \lesseqgtr 0, & (30b)\\
c^T_{\tau^c} &= -\sigma_c \frac{c^T}{(1+\tau^c)} < 0, & (30c)\\
c^N_{\bar{\lambda}} &= -\sigma_c \frac{c^N}{\bar{\lambda}} < 0, & (30d)\\
c^N_p &= -\frac{c^N}{p}\left[\left(1-\alpha_c\right)\phi + \alpha_c \sigma_c\right] < 0, & (30e)\\
c^N_{\tau^c} &= -\sigma_c \frac{c^N}{(1+\tau^c)} < 0, & (30f)
\end{aligned}$$

where we used the fact that $-\frac{p_c'' p}{p_c'} = \phi\left(1-\alpha_c\right) > 0$ and $p_c' c = c^N$.

A.2 Short-Run Static Solutions for Production-Side

Capital-Labor Ratios

¿From static optimality conditions (6a) and (6b), we may express sector capital-labor ratios as functions of the real exchange rate:

$$k^T = k^T\left(p, \mu\right), \qquad k^N = k^N\left(p, \mu\right), \tag{31}$$

with

$$k_p^T = \frac{\partial k^T}{\partial p} = \frac{h}{\mu f_{kk}\left(k^N - k^T\right)}, \tag{32a}$$

$$k_\mu^T = \frac{\partial k^T}{\partial \mu} = -\frac{ph}{(\mu)^2 f_{kk}\left(k^N - k^T\right)}, \tag{32b}$$

$$k_p^N = \frac{\partial k^N}{\partial p} = \frac{\mu f}{p^2 h_{kk}\left(k^N - k^T\right)}. \tag{32c}$$

$$k_\mu^N = \frac{\partial k^N}{\partial \mu} = -\frac{f}{p h_{kk}\left(k^N - k^T\right)}. \tag{32d}$$

Wage

Equality $\left[f\left(k^T\right) - k^T f_k\left(k^T\right)\right] \equiv w^F$ can be solved for the wage rate:

$$w = w\left(p, \tau^F, \mu\right), \tag{33}$$

with

$$w_p = \frac{\partial w}{\partial p} = -\frac{k^T f_{kk} k_p^T}{\left(1+\tau^F\right)} = -\frac{k^T}{\left(1+\tau^F\right)} \frac{h}{\mu\left(k^N - k^T\right)} \lesseqgtr 0, \tag{34a}$$

$$w_{\tau^F} = \frac{\partial w}{\partial \tau^F} = -\frac{w}{\left(1+\tau^F\right)} < 0, \tag{34b}$$

$$w_\mu = -\frac{\partial w}{\partial \mu} = -\frac{k^T f_{kk} k_\mu^T}{\left(1+\tau^F\right)} = \frac{k^T}{\left(1+\tau^F\right)} \frac{ph}{(\mu)^2\left(k^N - k^T\right)} \gtreqless 0. \tag{34c}$$

Labor

Substituting short-run static solutions for labor (27) and capital-labor ratios (31) into the resource constraints for capital and labor (7), we can solve for traded and non traded labor as follows:

$$L^T = L^T\left(K, p, \bar{\lambda}, \tau^F, \tau^H, \mu\right), \quad L^N = L^N\left(K, p, \bar{\lambda}, \tau^F, \tau^H, \mu\right), \tag{35}$$

with

$$
\begin{aligned}
L_K^T &= \frac{\partial L^T}{\partial K} = \frac{1}{k^T - k^N} \lesseqgtr 0, && \text{(36a)}\\
L_p^T &= \frac{\partial L^T}{\partial p} = \frac{1}{\mu\left(k^N - k^T\right)^2}\left[\frac{L^T h}{f_{kk}} + \frac{\mu^2 L^N f}{p^2 h_{kk}} - \sigma_L L \frac{\Lambda}{w^F} k^T k^N h\right] < 0, && \text{(36b)}\\
L_\mu^T &= \frac{\partial L^T}{\partial \mu} = -\frac{1}{\left[\mu\left(k^N - k^T\right)\right]^2}\left[\frac{L^T ph}{f_{kk}} + \frac{\mu^2 L^N f}{p h_{kk}} - \sigma_L L \frac{\Lambda}{w^F} k^T k^N ph\right] > 0, && \text{(36c)}\\
L_{\bar{\lambda}}^T &= \frac{\partial L^T}{\partial \bar{\lambda}} = \sigma_L \frac{L}{\bar{\lambda}} \frac{k^N}{k^N - k^T} \gtreqless 0, && \text{(36d)}\\
L_{\tau^F}^T &= \frac{\partial L^T}{\partial \tau^F} = -\frac{k^N}{k^N - k^T} \sigma_L L \frac{\Lambda}{\left(1+\tau^F\right)} \lesseqgtr 0, && \text{(36e)}\\
L_{\tau^H}^T &= \frac{\partial L^T}{\partial \tau^H} = -\frac{k^N}{k^N - k^T} \sigma_L L \frac{(w-\kappa)}{w^A} \lesseqgtr 0, && \text{(36f)}\\
L_K^N &= \frac{\partial L^N}{\partial K} = \frac{1}{k^N - k^T} \gtreqless 0, && \text{(36g)}\\
L_p^N &= \frac{\partial L^N}{\partial p} = -\frac{1}{\mu\left(k^N - k^T\right)^2}\left[\frac{L^T h}{f_{kk}} + \frac{\mu^2 L^N f}{p^2 h_{kk}} - \sigma_L L \frac{\Lambda}{w^F} \left(k^T\right)^2 h\right] > 0, && \text{(36h)}\\
L_\mu^N &= \frac{\partial L^N}{\partial \mu} = \frac{1}{\left[\mu\left(k^N - k^T\right)\right]^2}\left[\frac{L^T ph}{f_{kk}} + \frac{\mu^2 L^N f}{p h_{kk}} - \sigma_L L \frac{\Lambda}{w^F} \left(k^T\right)^2 ph\right] < 0, && \text{(36i)}\\
L_{\bar{\lambda}}^N &= \frac{\partial L^N}{\partial \bar{\lambda}} = -\sigma_L \frac{L}{\bar{\lambda}} \frac{k^T}{k^N - k^T} \lesseqgtr 0, && \text{(36j)}\\
L_{\tau^F}^N &= \frac{\partial L^T}{\partial \tau^F} = \frac{k^T}{k^N - k^T} \sigma_L L \frac{\Lambda}{\left(1+\tau^F\right)} \gtreqless 0, && \text{(36k)}\\
L_{\tau^H}^N &= \frac{\partial L^N}{\partial \tau^H} = \frac{k^T}{k^N - k^T} \sigma_L L \frac{(w-\kappa)}{w^A} \gtreqless 0, && \text{(36l)}
\end{aligned}
$$

where $w^F = w\left(1+\tau^F\right)$. From (36a) and (36g), when the capital stock rises, labor must shift to the sector which is relatively more capital intensive. From (36b) and (36h), a rise in the relative price of non tradable goods (lowers the capital-labor ratios) causes a shift of labor from the traded to the non-traded sector, irrespective of the sectoral capital intensities. From (36d) and (36j), an increase in the marginal utility of wealth raises the labor supplied by households which leads to a shift of labor to the sector which is relatively less capital capital intensive. From (36e)-(36f) and (36k)-(36l), a rise in the tax rate τ^F paid by firms or an increase in the tax rate τ^H paid by households reduces unambiguously total employment L which leads to a shift of labor towards the sector which is relatively more capital intensive.

Output

Inserting short-run static solutions for capital-labor ratios (31) and for labor (36) into the production functions, we can solve for the traded, $Y^T = L^T k^T$, and the non traded output, $Y^N = L^N h^N$:

$$
Y^T = Y^T\left(K, p, \bar{\lambda}, \tau^F, \tau^H, \mu\right), \qquad Y^N = Y^N\left(K, p, \bar{\lambda}, \tau^F, \tau^H, \mu\right), \tag{37}
$$

with

$$Y_K^T = \frac{\partial Y^T}{\partial K} = -\frac{f}{k^N - k^T} \lesseqgtr 0, \tag{38a}$$

$$Y_p^T = \frac{\partial Y^T}{\partial p} = \frac{1}{\mu\left(k^N - k^T\right)^2}\left[\frac{pL^T\left(h\right)^2}{\mu f_{kk}} + \frac{L^N\left(\mu f\right)^2}{\left(p\right)^2 h_{kk}} - \sigma_L L \frac{\Lambda}{w^F} k^T k^N h f\right] < 0, \tag{38b}$$

$$Y_\mu^T = \frac{\partial Y^T}{\partial \mu} = -\frac{1}{\left[\mu\left(k^N - k^T\right)\right]^2}\left[\frac{L^T\left(ph\right)^2}{\mu f_{kk}} + \frac{L^N\left(\mu f\right)^2}{p h_{kk}} - \sigma_L L \frac{\Lambda}{w^F} k^T k^N p h f\right] > 0, \tag{38c}$$

$$Y_{\bar{\lambda}}^T = \frac{\partial Y^T}{\partial \bar{\lambda}} = \sigma_L \frac{L}{\bar{\lambda}} \frac{k^N f}{k^N - k^T} \gtreqless 0, \tag{38d}$$

$$Y_{\tau^F}^T = \frac{\partial Y^T}{\partial \tau^F} = -\frac{k^N f}{k^N - k^T} \sigma_L L \frac{\Lambda}{\left(1 + \tau^F\right)} \lesseqgtr 0, \tag{38e}$$

$$Y_{\tau^H}^T = \frac{\partial Y^T}{\partial \tau^H} = -\frac{k^N f}{k^N - k^T} \sigma_L L \frac{\left(w - \kappa\right)}{w^A} \lesseqgtr 0, \tag{38f}$$

$$Y_K^N = \frac{\partial Y^N}{\partial K} = \frac{h}{k^N - k^T} \gtreqless 0, \tag{38g}$$

$$Y_p^N = \frac{\partial Y^N}{\partial p} = -\frac{1}{p\left(k^N - k^T\right)^2}\left[\frac{pL^T\left(h\right)^2}{\mu f_{kk}} + \frac{L^N\left(\mu f\right)^2}{p^2 h_{kk}} - \frac{p}{\mu}\sigma_L L \frac{\Lambda}{w^F}\left(k^T h\right)^2\right] > 0. \tag{38h}$$

$$Y_\mu^N = \frac{\partial Y^N}{\partial \mu} = \frac{1}{\mu\left(k^N - k^T\right)^2}\left[\frac{pL^T\left(h\right)^2}{\mu f_{kk}} + \frac{L^N\left(\mu f\right)^2}{p^2 h_{kk}} - \frac{p}{\mu}\sigma_L L \frac{\Lambda}{w^F}\left(k^T h\right)^2\right] < 0, \tag{38i}$$

$$Y_{\bar{\lambda}}^N = \frac{\partial Y^N}{\partial \bar{\lambda}} = -\sigma_L \frac{L}{\bar{\lambda}} \frac{k^T h}{k^N - k^T} \lesseqgtr 0, \tag{38j}$$

$$Y_{\tau^F}^N = \frac{\partial Y^N}{\partial \tau^F} = \frac{k^T h}{k^N - k^T} \sigma_L L \frac{\Lambda}{\left(1 + \tau^F\right)} \gtreqless 0, \tag{38k}$$

$$Y_{\tau^H}^N = \frac{\partial Y^N}{\partial \tau^H} = \frac{k^T h}{k^N - k^T} \sigma_L L \frac{\left(w - \kappa\right)}{w^A} \gtreqless 0. \tag{38l}$$

¿From (38b) and (38h), an appreciation in the real exchange rate attracts resources from the traded to the non traded sector which in turn raises the output of the latter. From (38a) and (38g), a rise in the capital stock raises the output of the sector which is relatively more capital intensive. From (38d) and (38j), an increase in the marginal utility of wealth lowers the output in the sector which is relatively less capital intensive. From (38e)-(38f) and (38k)-(38l), higher wage taxes, whatever they are levied on employers or employees depress total employment and favor the output in the sector which is relatively more capital intensive. As it will be useful to calculate fiscal multipliers, we give the partial derivatives of output in the traded and the non traded sector w. r. t. total employment:

$$Y_L^T = \frac{\partial Y^T}{\partial L} = \frac{k^N f}{k^N - k^T} \gtreqless 0, \quad Y_L^N = \frac{\partial Y^N}{\partial L} = -\frac{k^T h}{k^N - k^T} \lesseqgtr 0. \tag{39}$$

Useful Properties

Making use of (38b) and (38h), (38a) and (38g), we deduce the following useful properties:

$$
\begin{aligned}
Y_p^T + p\frac{Y_p^N}{\mu} &= -\sigma_L L\Lambda \frac{k^T h}{\mu\left(k^N - k^T\right)} \lesseqgtr 0, & \text{(40a)}\\
\mu Y_K^T + pY_K^N &= \frac{\mu f - ph}{k^T - k^N} = ph_k = \mu f_k, & \text{(40b)}\\
Y_L^T + p\frac{Y_L^N}{\mu} &= w^F, & \text{(40c)}\\
Y_\mu^T + p\frac{Y_\mu^N}{\mu} &= \sigma_L L\Lambda k^T \frac{ph}{\mu^2\left(k^N - k^T\right)} \gtreqless 0, & \text{(40d)}\\
Y_{\bar{\lambda}}^T + p\frac{Y_{\bar{\lambda}}^N}{\mu} &= \sigma_L \frac{L}{\bar{\lambda}} \frac{\left(k^N \mu f - k^T ph\right)}{\mu\left(k^N - k^T\right)} = \sigma_L \frac{L}{\bar{\lambda}} w^F > 0, & \text{(40e)}\\
Y_{\tau^F}^T + p\frac{Y_{\tau^F}^N}{\mu} &= -\sigma_L L w\Lambda < 0, & \text{(40f)}\\
Y_{\tau^H}^T + p\frac{Y_{\tau^H}^N}{\mu} &= -\sigma_L L\frac{(w-\kappa)}{w^A} w^F < 0, & \text{(40g)}
\end{aligned}
$$

where we used the fact that $\mu f \equiv p\left[h - h_k\left(k^N - k^T\right)\right]$ and $k^N \mu f - k^T ph = p\left(h - h^K k^N\right)\left(k^N - k^T\right) = \mu w^F\left(k^N - k^T\right)$.

In addition, using the fact that $r^K = f_k\left[k^T(p,\mu)\right]$, the rental rate of capital denoted by r^K can be expressed as a function of the real exchange rate p and the mark-up μ:

$$
r^K = r^K(p,\mu), \tag{41}
$$

with partial derivatives given by:

$$
\begin{aligned}
r_p^K &\equiv \frac{\partial r^K}{\partial p} = \frac{h}{\mu\left(k^N - k^T\right)} \gtreqless 0, & \text{(42a)}\\
r_p^\mu &\equiv \frac{\partial r^K}{\partial \mu} = -\frac{ph}{\mu^2\left(k^N - k^T\right)} \lesseqgtr 0. & \text{(42b)}
\end{aligned}
$$

B Equilibrium Dynamics and Formal Solutions

Inserting short-run static solutions (27), (29) and (37) into (26d) and (9), we obtain:

$$
\begin{aligned}
\dot{K} &= \frac{1}{\mu} Y^N\left(K, p, \bar{\lambda}, \tau^F, \tau^H\right) - c^N\left(\bar{\lambda}, p, \tau^c\right) - \delta_K K - g^N, & \text{(43a)}\\
\dot{p} &= p\left[r^\star + \delta_K - \frac{h_k(p)}{\mu}\right]. & \text{(43b)}
\end{aligned}
$$

Linearizing these two equations around the steady-state, and denoting $\tilde{x} = \tilde{K}, \tilde{p}$ the long-term values of $x = K, p$, we obtain in a matrix form:

$$
\left(\dot{K}, \dot{p}\right)^T = J\left(K(t) - \tilde{K}, p(t) - \tilde{p}\right)^T, \tag{44}
$$

where J is given by

$$
J \equiv \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \tag{45}
$$

with

$$b_{11} = \frac{Y_K^N}{\mu} - \delta_K = \frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}_T\right)} - \delta_K \gtrless 0, \quad b_{12} = \frac{Y_p^N}{\mu} - c_p^N > 0, \tag{46a}$$

$$b_{21} = 0, \quad b_{22} = -\tilde{p}\frac{h_{kk}k_p^N}{\mu} = -\frac{\tilde{f}}{\tilde{p}\left(\tilde{k}^N - \tilde{k}^T\right)} = \frac{Y_K^T}{\tilde{p}} \lessgtr 0. \tag{46b}$$

Equilibrium Dynamics

By denoting ν the eigenvalue of matrix J, the characteristic equation for the matrix of the linearized system (44) can be written as follows:

$$\nu^2 - \frac{1}{\tilde{p}}\left(Y_K^T + \frac{\tilde{p}}{\tilde{\mu}}Y_K^N - \delta_K\tilde{p}\right)\nu + \frac{Y_K^T}{\tilde{p}}\left(\frac{Y_K^N}{\mu} - \delta_K\right) = 0. \tag{47}$$

The determinant denoted by Det of the linearized 2×2 matrix (44) is unambiguously negative:[32]

$$\text{Det J} = b_{11}b_{22} = \frac{Y_K^T}{\tilde{p}}\left(\frac{Y_K^N}{\mu} - \delta_K\right) = -\frac{\tilde{f}\tilde{h}}{\mu\tilde{p}\left(\tilde{k}^N - \tilde{k}^T\right)^2} - \delta_K\frac{Y_K^T}{\tilde{p}} < 0, \tag{48}$$

and the trace denoted by Tr given by

$$\text{Tr J} = b_{11} + b_{22} = \frac{1}{\tilde{p}}\left(Y_K^T + \frac{\tilde{p}}{\tilde{\mu}}Y_K^N\right) - -\delta_K = \frac{h_k}{\mu} - \delta_K = r^\star > 0, \tag{49}$$

where we used the fact that at the long-run equilibrium $\frac{h_k}{\mu} = r^\star + \delta_K$.

¿From (44), the characteristic root obtained from J writes as follows:

$$\nu_i \equiv \frac{1}{2}\left\{r^\star \pm \sqrt{(r^\star)^2 - 4\frac{Y_K^T}{\tilde{p}}\left(\frac{Y_K^N}{\mu} - \delta_K\right)}\right\} \gtrless 0, \quad i = 1, 2. \tag{50}$$

Using (49), then (50) can be rewritten as follows:

$$\nu_i \equiv \frac{1}{2}\left\{r^\star \pm \left[\frac{Y_K^T}{\tilde{p}} - \left(\frac{Y_K^N}{\mu} - \delta_K\right)\right]\right\} \gtrless 0, \quad i = 1, 2. \tag{51}$$

We denote by $\nu_1 < 0$ and $\nu_2 > 0$ the stable and unstable real eigenvalues, satisfying

$$\nu_1 < 0 < r^\star < \nu_2. \tag{52}$$

Since the system features one state variable, K, and one jump variable, p, the equilibrium yields a unique one-dimensional stable saddle-path.

Formal Solutions

General solutions paths are given by :

$$K(t) - \tilde{K} = B_1 e^{\nu_1 t} + B_2 e^{\nu_2 t}, \tag{53a}$$

$$p(t) - \tilde{p} = \omega_2^1 B_1 e^{\nu_1 t} + \omega_2^2 B_2 e^{\nu_2 t}, \tag{53b}$$

[32] Starting from the equality of labor marginal products between sectors, using the fact that $f_k = ph_k$ and $h_k = r^\star + \delta_K$, it is straightforward to prove that b_{11} is positive in the case $k^N > k^T$.

where we normalized ω_1^i to unity. The eigenvector ω_2^i associated with eigenvalue μ_i is given by

$$\omega_2^i = \frac{\nu_i - b_{11}}{b_{12}}, \tag{54}$$

with

$$b_{11} = \frac{Y_K^N}{\mu} - \delta_K = \frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}_T\right)} - \delta_K \gtrless 0, \tag{55a}$$

$$b_{12} = \frac{Y_p^N}{\mu} - c_p^N > 0, \tag{55b}$$

where c_p^N is given by (30e).

Case $k^N > k^T$

This assumption reflects the fact that the capital-labor ratio of the non traded good sector exceeds the capital-labor of the traded sector. From (51), the stable and unstable eigenvalues can be rewritten as follows:

$$\nu_1 = -\frac{\tilde{f}}{\tilde{p}\left(\tilde{k}^N - \tilde{k}^T\right)} < 0, \tag{56a}$$

$$\nu_2 = \frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}^T\right)} - \delta_K > 0, \tag{56b}$$

since we suppose that $k^N > k^T$.

We can deduce the signs of several useful expressions:

$$Y_K^N = \mu\left(\nu_2 + \delta_K\right) > 0, \tag{57a}$$

$$Y_K^T = \tilde{p}\nu_1 < 0, \tag{57b}$$

$$\frac{\tilde{p}h_{kk}k_p^N}{\mu} = -\nu_1 > 0, \tag{57c}$$

$$Y_{\tau^F}^N = \tilde{k}^T\left(\nu_2 + \delta_K\right)\sigma_L\tilde{L}\frac{\tilde{\Lambda}}{\left(1+\tau^F\right)} > 0, \tag{57d}$$

$$Y_{\tau^F}^T = \tilde{p}\tilde{k}^N\nu_1\sigma_L\tilde{L}\frac{\tilde{\Lambda}}{\left(1+\tau^F\right)} < 0, \tag{57e}$$

$$Y_{\bar{\lambda}}^N = -\frac{1}{\bar{\lambda}}\sigma_L\tilde{L}\tilde{k}^T\mu\left(\nu_2 + \delta_K\right) < 0, \tag{57f}$$

$$Y_{\bar{\lambda}}^T = -\frac{1}{\bar{\lambda}}\sigma_L\tilde{L}\tilde{p}\tilde{k}^N\nu_1 > 0. \tag{57g}$$

We write out eigenvector ω^i, corresponding with stable eigenvalue ν_1 with $i = 1, 2$, to determine their signs:

$$\omega^1 = \begin{pmatrix} 1 & (+) \\ \frac{\nu_1 - \nu_2}{\left(\frac{Y_p^N}{\mu} - c_p^N\right)} & (-) \end{pmatrix}, \quad \omega^2 = \begin{pmatrix} 1 & (+) \\ 0 & \end{pmatrix}. \tag{58}$$

Case $k^T > k^N$

This assumption reflects the fact that the capital-labor ratio of the traded good sector exceeds the capital-labor ratio of the non traded sector. From (51), the stable and unstable eigenvalues can be rewritten as follows:

$$\nu_1 = \frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}^T\right)} - \delta_K < 0, \tag{59a}$$

$$\nu_2 = -\frac{\tilde{f}}{\tilde{p}\left(\tilde{k}^N - \tilde{k}^T\right)} > 0, \tag{59b}$$

since we suppose that $k^T > k^N$.

We can deduce the signs of several useful expressions:

$$Y_K^N = \mu\left(\nu_1 + \delta_K\right) < 0, \tag{60a}$$

$$Y_K^T = \tilde{p}\nu_2 > 0, \tag{60b}$$

$$\frac{\tilde{p}h_{kk}k_p^N}{\mu} = -\nu_2 < 0, \tag{60c}$$

$$Y_{\tau^F}^N = \tilde{k}^T\mu\left(\nu_1 + \delta_K\right)\sigma_L\tilde{L}\frac{\tilde{\Lambda}}{\left(1+\tau^F\right)} < 0, \tag{60d}$$

$$Y_{\tau^F}^T = \tilde{p}\tilde{k}^N\nu_2\sigma_L\tilde{L}\frac{\tilde{\Lambda}}{\left(1+\tau^F\right)} > 0, \tag{60e}$$

$$Y_{\bar{\lambda}}^N = -\frac{1}{\bar{\lambda}}\sigma_L\tilde{L}\tilde{k}^T\mu\left(\nu_1 + \delta_K\right) > 0, \tag{60f}$$

$$Y_{\bar{\lambda}}^T = -\frac{1}{\bar{\lambda}}\sigma_L\tilde{L}\tilde{p}\tilde{k}^N\nu_2 < 0. \tag{60g}$$

We write out the four eigenvectors ω^i, corresponding with stable eigenvalues ν_i with $i = 1, 2$, to determine their signs:

$$\omega^1 = \begin{pmatrix} 1 & (+) \\ 0 & \end{pmatrix}, \quad \omega^2 = \begin{pmatrix} 0 & \\ \frac{\nu_2 - \nu_1}{\left(\frac{Y_p^N}{\mu} - c_p^N\right)} & (+) \end{pmatrix}. \tag{61}$$

Formal Solution for the Stock of Foreign Assets

We first linearize equation (11) around the steady-state:

$$\dot{b}(t) = r^{\star}\left(b(t) - \tilde{b}\right) + Y_K^T\left(K(t) - \tilde{K}\right) + \left[Y_p^T - c_p^T\right]\left(p(t) - \tilde{p}\right). \tag{62}$$

where c_p^T is given by (30b).

Inserting general solutions for $K(t)$ and $p(t)$, the solution for the stock of international assets writes as follows:

$$\dot{b}(t) = r^{\star}\left(b(t) - \tilde{b}\right) + Y_K^T\sum_{i=1}^{2}B_i e^{\nu_i t} + \left[Y_p^T - c_p^T\right]\sum_{i=1}^{2}B_i\omega_2^i e^{\nu_i t}. \tag{63}$$

Solving the differential equation leads to the following expression:

$$b(t) - \tilde{b} = \left[\left(b_0 - \tilde{b}\right) - \Phi_1 B_1 - \Phi_2 B_2\right]e^{r^{\star}t} + \Phi_1 B_1 e^{\nu_1 t} + \Phi_2 B_2 e^{\nu_2 t}, \tag{64}$$

with

$$\Phi_i = \frac{N_i}{\nu_i - r^\star} = \frac{Y_K^T + \left[Y_p^T - c_p^T\right]\omega_2^i}{\nu_i - r^\star}, \quad i = 1, 2. \tag{65}$$

Invoking the transversality condition for intertemporal solvency, the terms in brackets of equation (64) must be null and we must set $B_2 = 0$. We obtain the linearized version of the nation's intertemporal budget constraint:

$$b_0 - \tilde{b} = \Phi_1\left(K_0 - \tilde{K}\right). \tag{66}$$

The stable solution for net foreign assets finally reduces to:

$$b(t) - \tilde{b} = \Phi_1\left(K(t) - \tilde{K}\right). \tag{67}$$

Case $k^N > k^T$

$$\begin{aligned} N_1 &= Y_K^T + \left(Y_p^T - c_p^T\right)\omega_2^1, \\ &= \tilde{p}\nu_2\left\{1 + \frac{\omega_2^1}{\tilde{p}\nu_2}\left[\sigma_c\tilde{c}^N - \sigma_L\tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\tilde{\Lambda}\right]\right\} \gtreqless 0, && \text{(68a)} \\ N_2 &= Y_K^T + \left(Y_p^T - c_p^T\right)\omega_2^2, && \text{(68b)} \\ &= Y_K^T = \tilde{p}\nu_1 < 0, && \text{(68c)} \end{aligned}$$

where (68c) follows from the fact that $\omega_2^2 = 0$. We made use of property (40a) together with the fact that $c_p^T = p_c c_p - p c_p^N$ to compute $Y_p^T - c_p^T = -\tilde{p}\left(\frac{Y_p^N}{\mu} - c_p^N\right) - p_c c_p - \sigma_L\tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\tilde{\Lambda} \gtreqless 0$.

The sign of Φ_1 is ambiguous and reflects the impact of the capital accumulation on the net foreign assets accumulation along a stable transitional path:

$$\dot{b}(t) = \Phi_1\dot{K}(t).$$

where $\dot{K}(t) = \nu_1 B_1 e^{\nu_1 t}$. Following empirical evidence suggesting that the current account and investment are negatively correlated (see e. g. Glick and Rogoff [1995]), we will impose thereafter:

Assumption 1 $\Phi_1 < 0$ *which implies that* $N_1 > 0$.

The condition for the assumption to hold, i. e. $N_1 > 0$, may be rewritten as follows:

$$\nu_2 > -\frac{\omega_2^1}{\tilde{p}}\left[\sigma_c\tilde{c}^N - \sigma_L\tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\tilde{\Lambda}\right]. \tag{69}$$

Case $k^T > k^N$

$$\begin{aligned} N_1 &= Y_K^T + \left(Y_p^T - c_p^T\right)\omega_2^1, \\ &= Y_K^T = \tilde{p}\nu_2 > 0, && \text{(70a)} \\ N_2 &= Y_K^T + \left(Y_p^T - c_p^T\right)\omega_2^2, \\ &= \tilde{p}\nu_1\left\{1 + \frac{\omega_2^2}{\tilde{p}\nu_1}\left[\sigma_c\tilde{c}^N - \sigma_L\tilde{L}\tilde{k}^T\left(\nu_1 + \delta_k\right)\tilde{\Lambda}\right]\right\}, \lesseqgtr 0, && \text{(70b)} \end{aligned}$$

where (70a) follows from the fact that $\omega_2^1 = 0$. We made use of property (40a) together with $c_p^T = p_c c_p - p c_p^N$ to compute $Y_p^T - c_p^T = -\tilde{p}\left(\frac{Y_p^N}{\mu} - c_p^N\right) - p_c c_p - \sigma_L \tilde{L} \tilde{k}^T \left(\nu_1 + \delta_K\right) \gtreqless 0$.

C Derivation of the Current Account Equation

This section is dedicated to the determination of current account law of motion. Substituting the definition of transfer Z by using (8), substituting the market clearing condition for non traded goods (9) into (3) we get:

$$\begin{aligned} \dot{b} &= r^\star b(t) + r^K K(t) + w^A L(t) - p_c\left(1+\tau^c\right) c(t) - p(t) I(t) + Z, \\ &= r^\star b + \left(r^K K + w^F L\right) - p_c c - p\left(\frac{Y^N}{\mu} - c^N - g^N\right). \end{aligned}$$

Using the fact that $L^T + L^N = L$, $K^T + K^N = K$, , the dynamic equation for the current account can be rewritten as follows:

$$\begin{aligned} \dot{b} &= r^\star b + \left[w^F L^T + \left(r^K\right) K^T\right] + \left[w^F L^N + \left(r^K + \delta_K\right) K^N\right] - p\frac{Y^N}{\mu} - c^T - g^T, \\ &= r^\star b + Y^T - c^T - g^T, \end{aligned}$$

where the overall variable cost $w^F L^N + r^K K^N$ in the non traded sector and output net of fixed cost in that sector, i. e. $p\frac{Y^N}{\mu} = pZ^N$, cancel each other.[33]

D Savings

Formal Solution for Financial Wealth

The law of motion for financial wealth ($S(t) = \dot{a}(t)$) is given by:

$$\dot{a}(t) = r^\star a(t) + \left[w\left(p, \tau^F, \eta\right)\left(1-\tau^H\right) + \tau^H \kappa\right] L\left(\bar{\lambda}, p, \tau^F, \tau^H\right) - p_c(p)\left(1+\tau^c\right) c\left(\bar{\lambda}, p, \tau^c\right) + Z, \tag{71}$$

with $Z = \tau^c p_c c + \left[\left(\tau^H + \tau^F\right) w - \tau^H \kappa\right] L - g^T - \tilde{p} g^N$.

The linearized version of (71) writes as follows:

$$\dot{a}(t) = r^\star\left(a(t) - \tilde{a}\right) + M_1\left(p(t) - \tilde{p}\right), \tag{72}$$

with M_1 given by

$$\begin{aligned} M_1 &= \left(w_p \tilde{L} + \tilde{w} L_p\right)\left(1+\tau^F\right) - \left(\tilde{c}^N + p_c c_p + g^N\right), \\ &= \left(1+\tau^F\right) \tilde{L} w_p\left(1 + \tilde{\Lambda}\sigma_L\right) - \left[\tilde{c}^N\left(1-\sigma_c\right) + g^N\right], \\ &= -\left\{\tilde{K}\left(\mu_2 + \delta_K\right) + \left[\sigma_L \tilde{L}\tilde{\Lambda}\tilde{k}^T\left(\mu_2 + \delta_K\right) - \sigma_c \tilde{c}^N\right]\right\} < 0. \end{aligned} \tag{73}$$

[33] In the traded sector which is perfectly competitive, we have : $Y^T = F_L L^T + r^K K^T = w^F L^T + r^K K^T$. Instead, in the non traded sector which is imperfectly competitive we have: $pZ^N = p\frac{H_L}{\mu} L^N + p\frac{H_K}{\mu} K^N$ or $p\mu Z^N = pY^N = pH_L L^N + pH_K K^N = w^F L^N + r^K K^N$.

¿From the second line of (73), if $\sigma_c < 1$ as empirical studies suggest, then the term in square brackets is positive and M_1 is negative. The last line has been computed by using the fact that $\tilde{L} = \tilde{L}^N + \tilde{L}^T$ and $\tilde{K} = \tilde{k}^T \tilde{L}^T + \tilde{k}^N \tilde{L}^N$ which allows to simplify $\frac{1}{\mu}\left[\tilde{Y}^N + \tilde{L}\tilde{k}^T\left(\mu_2 + \delta_K\right)\mu\right]$ to $\tilde{K}\left(\mu_2 + \delta_K\right)$.

The general solution for the stock of financial wealth writes as follows:

$$a(t) = \tilde{a} + \left[(a_0 - \tilde{a}) - \frac{M_1 \omega_2^1}{\mu_1 - r^\star} B_1\right] e^{r^\star t} + \frac{M_1 \omega_2^1}{\mu_1 - r^\star} B_1 e^{\mu_1 t}, \tag{74}$$

where we used the fact that $\omega_2^2 = 0$.

Invoking the transversality condition, we obtain the stable solution for financial wealth:

$$a(t) = \tilde{a} + \frac{M_1 \omega_2^1}{\mu_1 - r^\star} B_1 e^{\mu_1 t}, \tag{75}$$

and the intertemporal solvency condition

$$\tilde{a} - a_0 = \frac{M_1 \omega_2^1}{\mu_1 - r^\star}\left(\tilde{K} - K_0\right). \tag{76}$$

Steady-State and Dynamic Effects of Tax Shocks

Differentiating (76) w. r. t. τ^j $(j = F, H)$, long-term changes of financial wealth are given by:

$$\frac{\mathrm{d}\tilde{a}}{\mathrm{d}\tau^j} = \frac{\omega_2^1}{\mu_2}\left(\tilde{K}\mu_2 + \sigma_L \tilde{L}\tilde{k}^T \mu_2 - \sigma_c \tilde{c}^N\right)\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j}. \tag{77}$$

Differentiating (75) w. r. t. τ^c and τ^j $(j = F, H)$, we get the dynamics of savings:

$$S(t) = \dot{a}(t) = \mu_1 \frac{M_1 \omega_2^1}{\mu_1 - r^\star}\frac{B_1}{\mathrm{d}\tau^c}\mathrm{d}\tau^c e^{\mu_1 t} > 0, \tag{78a}$$

$$S(t) = \dot{a}(t) = \mu_1 \frac{M_1 \omega_2^1}{\mu_1 - r^\star}\frac{B_1}{\mathrm{d}\tau^j}\mathrm{d}\tau^j e^{\mu_1 t} < 0, \quad j = F, H, \tag{78b}$$

where $\frac{B_1}{\mathrm{d}\tau^c} = -\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^c} < 0$ and $\frac{B_1}{\mathrm{d}\tau^j} = -\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j} > 0$ as we shall see now in the next section.

E Long-Run Effects of Labor and Consumption Tax Changes

In this section, we calculate formal expressions of steady-state changes. For clarity purpose, we assume that $\delta_K = 0$ since it does not modify qualitatively the long-run effects of tax policies. This assumption will be relaxed in numerical analysis. We totally differentiate the steady-state which yields in a matrix form:

$$\begin{pmatrix} \frac{h_{kk}k_p^N}{\mu} & 0 & 0 & 0 \\ \left(\frac{Y_p^N}{\mu} - c_p^N\right) & \frac{Y_K^N}{\mu} & \left(\frac{Y_{\bar{\lambda}}^N}{\mu} - c_{\bar{\lambda}}^N\right) & 0 \\ \left(Y_p^T - c_p^T\right) & Y_K^T & \left(Y_{\bar{\lambda}}^T - c_{\bar{\lambda}}^T\right) & r^\star \\ 0 & -\Phi_1 & 0 & 1 \end{pmatrix}\begin{pmatrix} \mathrm{d}\tilde{p} \\ \mathrm{d}\tilde{K} \\ \mathrm{d}\bar{\lambda} \\ \mathrm{d}\tilde{b} \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{Y_{\tau^F}^N}{\mu}\mathrm{d}\tau^F - \frac{Y_{\tau^H}^N}{\mu}\mathrm{d}\tau^H + c_{\tau^c}^N \mathrm{d}\tau^c \\ -Y_{\tau^F}^T \mathrm{d}\tau^F - Y_{\tau^H}^T \mathrm{d}\tau^H + c_{\tau^c}^T \mathrm{d}\tau^c \\ 0 \end{pmatrix} \tag{79}$$

The determinant denoted by D of the matrix of coefficients is given by:

$$D \equiv \frac{h_{kk}k_p^N}{\mu}\left\{\frac{Y_K^N}{\mu}\left(Y_{\bar{\lambda}}^T - c_{\bar{\lambda}}^T\right) - \left(\frac{Y_{\bar{\lambda}}^N}{\mu} - c_{\bar{\lambda}}^N\right)\left[Y_K^T + r^\star \Phi_1\right]\right\} \tag{80}$$

We have to consider two cases, depending on wether the non traded sector is more or less capital intensive than the traded sector:

$$D = -\frac{\nu_1\nu_2}{\tilde{p}\bar{\lambda}}\left(\sigma_L \tilde{w}^F \tilde{L} + \sigma_c p_c \tilde{c}\right) > 0, \quad \text{case} \quad k^T > k^N, \tag{81a}$$

$$D = -\frac{\nu_1\nu_2}{\tilde{p}\bar{\lambda}}\left\{\left(\sigma_L \tilde{w}^F \tilde{L} + \sigma_c p_c \tilde{c}\right) + \frac{r^\star}{\nu_2}\frac{\omega_2^1}{\nu_2}\left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L}\tilde{k}^T \nu_2\right)\left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L}\tilde{k}^T \nu_2 \tilde{\Lambda}\right)\right\} > 0, \quad \text{case} \quad k^N > k^T, \tag{81b}$$

where we used the fact that and $\mu f k^N - phk^T = \mu w^F \left(k^N - k^T\right)$ together with $-p\left(k^N\nu_2 + k^T\nu_1\right) \equiv w^F$ if $k^T > k^N$ or $-p\left(k^N\nu_1 + k^T\nu_2\right) \equiv w^F$ if $k^N > k^T$.

Useful Expressions

We have computed these useful expressions:

$$\frac{Y_K^N}{\mu}Y_{\bar{\lambda}}^T - Y_K^T\frac{Y_{\bar{\lambda}}^N}{\mu} = \sigma_L \frac{\tilde{L}}{\bar{\lambda}}\frac{\tilde{h}\tilde{f}}{\left(\tilde{k}^N - \tilde{k}^T\right)}, \tag{82a}$$

$$p_c' Y_K^T - (1-\alpha_c)\, p_c \frac{Y_K^N}{\mu} = -\frac{p_c}{\tilde{p}}\left[\frac{\alpha_c \tilde{f} + (1-\alpha_c)\,\tilde{p}\tilde{h}}{\left(\tilde{k}^N - \tilde{k}^T\right)}\right], \tag{82b}$$

$$\frac{Y_{\bar{\lambda}}^N}{\mu} - c_{\bar{\lambda}}^N = \frac{1}{\bar{\lambda}}\left[-\sigma_L \tilde{L}\tilde{k}^T \frac{\tilde{h}}{\left(\tilde{k}^N - \tilde{k}^T\right)} + \sigma_c \tilde{c}^N\right], \tag{82c}$$

$$Y_K^T - c_{\bar{\lambda}}^T = \frac{\tilde{p}}{\bar{\lambda}}\left[\sigma_L \tilde{L}\tilde{k}^N \frac{\tilde{f}}{\tilde{p}\left(\tilde{k}^N - \tilde{k}^T\right)} + \sigma_c \frac{\tilde{c}^T}{\tilde{p}}\right]. \tag{82d}$$

In the case $\underline{k^N > k^T}$, useful expressions (89) write as follows:

$$\frac{Y_K^N}{\mu}Y_{\bar{\lambda}}^T - Y_K^T\frac{Y_{\bar{\lambda}}^N}{\mu} = -\tilde{p}\nu_1\nu_2\frac{\sigma_L \tilde{L}}{\bar{\lambda}}\left(\tilde{k}^N - \tilde{k}^T\right) > 0, \tag{83a}$$

$$p_c' Y_K^T - (1-\alpha_c)\, p_c \frac{Y_K^N}{\mu} = -p_c\left[\nu_2 - \alpha_c r^\star\right] < 0, \tag{83b}$$

$$\frac{Y_{\bar{\lambda}}^N}{\mu} - c_{\bar{\lambda}}^N = -\frac{1}{\bar{\lambda}}\left[\sigma_L \tilde{L}\tilde{k}^T \nu_2 - \sigma_c \tilde{c}^N\right] < 0, \tag{83c}$$

$$Y_{\bar{\lambda}}^T - c_{\bar{\lambda}}^T = -\frac{1}{\bar{\lambda}}\left[\sigma_L \tilde{L}\tilde{p}\tilde{k}^N \nu_1 - \sigma_c \tilde{c}^T\right] > 0, \tag{83d}$$

$$\frac{Y_K^N}{\mu}c_{\tau^c}^T - Y_K^T c_{\tau^c}^N = \frac{\sigma_c p_c \tilde{c}}{(1+\tau^c)}\left[-\nu_2(1-\alpha_c) + \nu_1\alpha_c\right] < 0, \tag{83e}$$

$$Y_K^T\frac{Y_{\tau^F}^N}{\mu} - \frac{Y_K^N}{\mu}Y_{\tau^F}^T = -\tilde{p}\nu_1\nu_2\sigma_L \tilde{L}\frac{\tilde{\Lambda}}{(1+\tau^F)}\left(\tilde{k}^N - \tilde{k}^T\right) > 0, \tag{83f}$$

$$Y_K^T\frac{Y_{\tau^H}^N}{\mu} - \frac{Y_K^N}{\mu}Y_{\tau^H}^T = -\tilde{p}\nu_1\nu_2\sigma_L \tilde{L}\frac{(\tilde{w}-\kappa)}{\tilde{w}^A}\left(\tilde{k}^N - \tilde{k}^T\right) > 0, \tag{83g}$$

$$Y_p^T - c_p^T = \left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L}\tilde{\Lambda}\tilde{k}^T \nu_2\right) - \tilde{p}\left(\frac{Y_p^N}{\mu} - c_p^N\right). \tag{83h}$$

where we used the fact that $Y_p^T = -\tilde{p}\frac{Y_p^N}{\mu} - \sigma_L \tilde{L}\tilde{\Lambda}\tilde{k}^T \nu_2$, $c_p^T = p_c c_p - \tilde{p} c_p^N$ and $p_c c_p = -\sigma_c \tilde{c}^N$ to rewrite $Y_p^T - c_p^T$ (see (83h)).

In the case $\underline{k^T > k^N}$, useful expressions (89) write as follows:

$$\begin{aligned}
\frac{Y_K^N}{\mu} Y_{\bar{\lambda}}^T - Y_K^T \frac{Y_{\bar{\lambda}}^N}{\mu} &= \tilde{p}\nu_1\nu_2 \frac{\sigma_L \tilde{L}}{\bar{\lambda}} \left(\tilde{k}^T - \tilde{k}^N\right) < 0, & \text{(84a)}\\
p_c' Y_K^T - (1-\alpha_c) p_c Y_K^N &= -p_c \left[\nu_1 - \alpha_c r^\star\right] > 0, & \text{(84b)}\\
\frac{Y_{\bar{\lambda}}^N}{\mu} - c_{\bar{\lambda}}^N &= -\frac{1}{\bar{\lambda}} \left(\sigma_L \tilde{L}\tilde{k}^T \nu_1 - \sigma_c \tilde{c}^N\right) > 0, & \text{(84c)}\\
Y_{\bar{\lambda}}^T - c_{\bar{\lambda}}^T &= -\frac{1}{\bar{\lambda}} \left[\sigma_L \tilde{L}\tilde{p}\tilde{k}^N \nu_2 - \sigma_c \tilde{c}^T\right] \gtrless 0, & \text{(84d)}\\
\frac{Y_K^N}{\mu} c_{\tau^c}^T - Y_K^T c_{\tau^c}^N &= \frac{\sigma_c p_c \tilde{c}}{(1+\tau^c)} \left[-\nu_1 (1-\alpha_c) + \nu_2 \alpha_c\right] > 0, & \text{(84e)}\\
Y_K^T \frac{Y_{\tau^F}^N}{\mu} - \frac{Y_K^N}{\mu} Y_{\tau^F}^T &= \tilde{p}\nu_1\nu_2 \sigma_L \tilde{L} \frac{\tilde{\Lambda}}{(1+\tau^F)} \left(\tilde{k}^T - \tilde{k}^N\right) < 0, & \text{(84f)}\\
Y_p^T - c_p^T &= \left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L}\tilde{\Lambda}\tilde{k}^T \nu_1\right) - \tilde{p}\left(\frac{Y_p^N}{\mu} - c_p^N\right), & \text{(84g)}\\
Y_K^T + r^\star \Phi_1 &= -\tilde{p}\nu_1. & \text{(84h)}
\end{aligned}$$

Long-Run Effects of an Unanticipated Permanent Change in the Consumption Tax Rate

case $k^N > k^T$

$$\begin{aligned}
\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^c} &= -\left(\frac{\sigma_c \tilde{c}\sigma_L \tilde{L}}{\Delta(1+\tau^c)}\right)\left[w^F - \tilde{k}^T \frac{r^\star}{\nu_2}\omega_2^1 \left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L} k^T \nu_2 \tilde{\Lambda}\right)\right] \lessgtr 0, & \text{(85a)}\\
\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^c} &= -\left(\frac{\sigma_c p_c \tilde{c}\sigma_L \tilde{L}}{1+\tau^c}\right)\frac{1}{\Delta}\left[1 + \alpha_c \frac{r^\star}{\nu_2}\frac{\omega_2^1}{\tilde{p}\nu_2}\left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L} k^T \nu_2 \tilde{\Lambda}\right)\right] < 0, & \text{(85b)}\\
\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^c} &= -\left(\frac{\sigma_c \bar{\lambda} p_c \tilde{c}}{1+\tau^c}\right)\frac{1}{\Delta}\left[1 + \alpha_c \frac{r^\star}{\nu_2}\frac{\omega_2^1}{\tilde{p}\nu_2}\left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L} k^T \nu_2 \tilde{\Lambda}\right)\right] < 0, & \text{(85c)}\\
\frac{\mathrm{d}\tilde{p}}{\mathrm{d}\tau^c} &= 0, & \text{(85d)}\\
\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^c} &= \frac{1}{\nu_2}\left(\frac{1}{1+\tau^c}\right)\left(\frac{\sigma_c p_c \tilde{c}\sigma_L \tilde{L}}{\Delta}\right)\left[\alpha_c \tilde{k}^N \nu_1 - (1-\alpha_c)\tilde{k}^T \nu_2\right] < 0, & \text{(85e)}\\
\frac{\mathrm{d}\tilde{b}}{\mathrm{d}\tau^c} &= \Phi_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^c} > 0, & \text{(85f)}
\end{aligned}$$

where $\Delta = \left[\left(\sigma_L \tilde{w}^F \tilde{L} + \sigma_c p_c \tilde{c}\right) + \frac{r^\star}{\nu_2}\frac{\omega_2^1}{\nu_2}\left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L}\tilde{k}^T \nu_2\right)\left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L}\tilde{k}^T \nu_2 \tilde{\Lambda}\right)\right]$ is assumed to be positive.

case $k^T > k^N$

$$\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^c} = -\left(\frac{1}{1+\tau^c}\right)\left(\frac{\sigma_c \tilde{c} \sigma_L \tilde{w}^F \tilde{L}}{\sigma_L \tilde{w}^F \tilde{L} + \sigma_c p_c \tilde{c}}\right) < 0, \tag{86a}$$

$$\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^c} = -\left(\frac{1}{1+\tau^c}\right)\left(\frac{\sigma_c p_c \tilde{c} \sigma_L \tilde{L}}{\sigma_L \tilde{w}^F \tilde{L} + \sigma_c p_c \tilde{c}}\right) < 0, \tag{86b}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^c} = -\bar{\lambda}\left(\frac{1}{1+\tau^c}\right)\left(\frac{\sigma_c p_c \tilde{c}}{\sigma_L \tilde{w}^F \tilde{L} + \sigma_c p_c \tilde{c}}\right) < 0, \tag{86c}$$

$$\frac{\mathrm{d}\tilde{p}}{\mathrm{d}\tau^c} = 0, \tag{86d}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^c} = \frac{1}{\nu_1}\left(\frac{1}{1+\tau^c}\right)\left(\frac{\sigma_c p_c \tilde{c} \sigma_L \tilde{L}}{\sigma_L \tilde{w}^F \tilde{L} + \sigma_c p_c \tilde{c}}\right)\left(\alpha_c \tilde{k}^N \nu_2 - (1-\alpha_c)\tilde{k}^T \nu_1\right) < 0, \tag{86e}$$

$$\frac{\mathrm{d}\tilde{b}}{\mathrm{d}\tau^c} = -\tilde{p}\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^c} > 0. \tag{86f}$$

Long-Run Effects of an Unanticipated Permanent Change in the Payroll Tax Rate

case $k^N > k^T$

$$\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^F} = -\sigma_c \tilde{c}\frac{\sigma_L \tilde{L}}{\Delta}\frac{\tilde{\Lambda}}{(1+\tau^F)}\left[\tilde{w}^F - \tilde{k}^T r^\star \frac{\omega_2^1}{\nu_2}\left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L} k^T \nu_2 \tilde{\Lambda}\right)\right] \lessgtr 0, \tag{87a}$$

$$\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^F} = -\frac{\sigma_L \tilde{L}}{\Delta}\frac{\tilde{\Lambda}}{(1+\tau^F)}\left\{\sigma_c p_c \tilde{c} + \frac{r^\star}{\nu_2}\frac{\omega_2^1}{\nu_2}\left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L} k^T \nu_2 \tilde{\Lambda}\right)\sigma_c \tilde{c}^N\right\} < 0, \tag{87b}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^F} = \bar{\lambda}\frac{\sigma_L \tilde{L}}{\Delta}\frac{\tilde{\Lambda}}{(1+\tau^F)}\left[\tilde{w}^F - \tilde{k}^T r^\star \frac{\omega_2^1}{\nu_2}\left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L} k^T \nu_2 \tilde{\Lambda}\right)\right] \lessgtr 0, \tag{87c}$$

$$\frac{\mathrm{d}\tilde{p}}{\mathrm{d}\tau^F} = 0, \tag{87d}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^F} = \frac{\sigma_L \tilde{L}}{\Delta \nu_2}\frac{\tilde{\Lambda}}{(1+\tau^F)}\sigma_c p_c \tilde{c}\left[\alpha_c \tilde{k}^N \nu_1 - (1-\alpha_c)\tilde{k}^T \nu_2\right] < 0, \tag{87e}$$

$$\frac{\mathrm{d}\tilde{b}}{\mathrm{d}\tau^F} = \Phi_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^F} > 0, \tag{87f}$$

where we used the fact that $\tilde{p}\left(\nu_2 \tilde{k}^T + \nu_1 \tilde{k}^N\right) = -\tilde{p}\left(\tilde{h} - h_k \tilde{k}^N\right) \equiv -\tilde{w}^F$ and $L_{\tau^H} = L_{\tau^F}\frac{(\tilde{w}-\kappa)}{\tilde{w}^A}\frac{1+\tau^F}{\tilde{\Lambda}}$.

case $k^T > k^N$

$$\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^F} = -\sigma_c\tilde{c}\frac{\sigma_L\tilde{L}}{\Delta}\frac{\tilde{\Lambda}}{(1+\tau^F)}\tilde{w}^F = \frac{1+\tau^c}{1+\tau^F}\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^c} < 0, \tag{88a}$$

$$\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^F} = -\frac{\sigma_L\tilde{L}}{\Delta}\frac{\tilde{\Lambda}}{(1+\tau^F)}\sigma_c p_c\tilde{c} = \frac{1+\tau^c}{1+\tau^F}\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^c} < 0, \tag{88b}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^F} = \bar{\lambda}\frac{\sigma_L\tilde{L}}{\Delta}\frac{\tilde{\Lambda}}{(1+\tau^F)}\tilde{w}^F > 0, \tag{88c}$$

$$\frac{\mathrm{d}\tilde{p}}{\mathrm{d}\tau^F} = 0, \tag{88d}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^F} = \frac{\sigma_L\tilde{L}}{\nu_1\Delta}\frac{\tilde{\Lambda}}{(1+\tau^F)}\sigma_c p_c\tilde{c}\left[\alpha_c\tilde{k}^N\nu_2 - (1-\alpha_c)\tilde{k}^T\nu_1\right] < 0, \tag{88e}$$

$$\frac{\mathrm{d}\tilde{b}}{\mathrm{d}\tau^F} = -\tilde{p}\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^F} > 0, \tag{88f}$$

where we let $\Delta \equiv \sigma_L\tilde{w}^F\tilde{L} + \sigma_c p_c\tilde{c}$ and we used the fact that $-\tilde{p}\left(\nu_1\tilde{k}^T + \nu_2\tilde{k}^N\right) = \tilde{p}\left(\tilde{h} - h_k\tilde{k}^N\right) \equiv \tilde{w}^F$.

Long-Run Effects of an Unanticipated Permanent Change in the Wage Income Tax Rate

case $k^N > k^T$

$$\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^H} = -\sigma_c\tilde{c}\frac{\sigma_L\tilde{L}}{\Delta}\frac{(\tilde{w}-\kappa)}{\tilde{w}^A}\left[\tilde{w}^F - \tilde{k}^T r^\star\frac{\omega_2^1}{\nu_2}\left(\sigma_c\tilde{c}^N - \sigma_L\tilde{L}k^T\nu_2\tilde{\Lambda}\right)\right] \lesseqgtr 0, \tag{89a}$$

$$\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^H} = -\frac{\sigma_L\tilde{L}}{\Delta}\frac{(\tilde{w}-\kappa)}{\tilde{w}^A}\left\{\sigma_c p_c\tilde{c} + \frac{r^\star}{\nu_2}\frac{\omega_2^1}{\nu_2}\left(\sigma_c\tilde{c}^N - \sigma_L\tilde{L}k^T\nu_2\tilde{\Lambda}\right)\sigma_c\tilde{c}^N\right\} < 0, \tag{89b}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^H} = \bar{\lambda}\frac{\sigma_L\tilde{L}}{\Delta}\frac{(\tilde{w}-\kappa)}{\tilde{w}^A}\left[\tilde{w}^F - \tilde{k}^T r^\star\frac{\omega_2^1}{\nu_2}\left(\sigma_c\tilde{c}^N - \sigma_L\tilde{L}k^T\nu_2\tilde{\Lambda}\right)\right] \lesseqgtr 0, \tag{89c}$$

$$\frac{\mathrm{d}\tilde{p}}{\mathrm{d}\tau^H} = 0, \tag{89d}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^H} = \frac{\sigma_L\tilde{L}}{\nu_2\Delta}\sigma_c p_c\tilde{c}\frac{(\tilde{w}-\kappa)}{\tilde{w}^A})\left[\alpha_c\tilde{k}^N\nu_1 - (1-\alpha_c)\tilde{k}^T\nu_2\right] < 0, \tag{89e}$$

$$\frac{\mathrm{d}\tilde{b}}{\mathrm{d}\tau^H} = -\Phi_1\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^H} > 0, \tag{89f}$$

where we used the fact that $\frac{\tilde{w}\left(1-\tau^H\right)}{\tilde{w}^A} = \tilde{\Lambda}$.

case $k^T > k^N$

$$\begin{aligned}
\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^H} &= -\sigma_c\tilde{c}\frac{\sigma_L\tilde{L}}{\Delta}\frac{(\tilde{w}-\kappa)}{\tilde{w}^A}\tilde{w}^F = \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^F}\frac{(\tilde{w}-\kappa)}{\tilde{w}^A}\frac{1+\tau^F}{\tilde{\Lambda}} < 0, && (90a)\\
\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^H} &= -\frac{\sigma_L\tilde{L}}{\Delta}\frac{(\tilde{w}-\kappa)}{\tilde{w}^A}\sigma_c p_c\tilde{c} < 0, && (90b)\\
\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^H} &= \bar{\lambda}\frac{\sigma_L\tilde{L}}{\Delta}\frac{(\tilde{w}-\kappa)}{\tilde{w}^A}\tilde{w}^F > 0, && (90c)\\
\frac{\mathrm{d}\tilde{p}}{\mathrm{d}\tau^H} &= 0, && (90d)\\
\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^H} &= \frac{\sigma_L\tilde{L}}{\nu_1\Delta}\frac{(\tilde{w}-\kappa)}{\tilde{w}^A}\sigma_c p_c\tilde{c}\left[\alpha_c\tilde{k}^N\nu_2-(1-\alpha_c)\,\tilde{k}^T\nu_1\right] < 0, && (90e)\\
\frac{\mathrm{d}\tilde{b}}{\mathrm{d}\tau^H} &= -\tilde{p}\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^H} > 0, && (90f)
\end{aligned}$$

where we let $\Delta = \sigma_L\tilde{w}^F\tilde{L} + \sigma_c p_c\tilde{c}$.

Inelastic Labor Supply Case: $\sigma_L = 0$

If labor is supplied inelastically, then the intertemporal elasticity of substitution for labor is null and total employment remains fixed to the level $\bar{L} = 1$.

Long-Run Effects of an Unanticipated Permanent Change of the Consumption Tax Rate

Set $\sigma_L = 0$ into (85) or (86), we get:

$$\begin{aligned}
\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^c} &= \frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^c} = \frac{\mathrm{d}\tilde{p}}{\mathrm{d}\tau^c} = \frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^c} = \frac{\mathrm{d}\tilde{b}}{\mathrm{d}\tau^c} = 0, && (91a)\\
\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^c} &= -\frac{\bar{\lambda}}{(1+\tau^c)}. && (91b)
\end{aligned}$$

¿From (91a)-(91b), the elasticity of the marginal utility of wealth is equal to unity in absolute terms and the long-run levels of variables remain unaffected. A rise in consumption tax raises the marginal cost of current consumption. Since the trade-off between labor and leisure turns out to be irrelevant, total employment remains fixed such that $\bar{\lambda}$ must fall by the same proportion than the rise in τ^c thus leaving unaffected real consumption as the *direct effect* and the *wealth effect* cancel each other. Since demand for non tradables and tradables remain unaffected, capital stock and net foreign assets must not change for investment and the current account to be zero in the long-run. As the capital stock remains unchanged in the long-run, dynamics degenerate.

Long-Run Effects of an Unanticipated Permanent Change of the Payroll Tax Rate

Set $\sigma_L = 0$ into (87) or (88), we get:

$$\begin{aligned}
\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^F} &= \frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^F} = \frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^F} = \frac{\mathrm{d}\tilde{p}}{\mathrm{d}\tau^F} = \frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^F} = \frac{\mathrm{d}\tilde{b}}{\mathrm{d}\tau^F} = 0, && (92a)\\
\frac{\mathrm{d}\tilde{w}}{\mathrm{d}\tau^F} &= -\frac{\tilde{w}}{(1+\tau^F)} < 0. && (92b)
\end{aligned}$$

¿From (92a)-(92b), a fall in τ^F leaves unchanged the steady-state levels of variables, and more importantly does no longer induce a *wealth effect*. The explanation is that whenever the trade-off between labor and leisure turns out to be irrelevant, total employment remains fixed. To insure that equality of sectoral labor marginal products holds, the wage must rise by the same proportion than the fall in the payroll tax. As the capital stock remains unchanged in the long-run, dynamics degenerate. In words, if labor is fixed, a change in the tax on wage paid by producers induces solely a *direct effect* on the wage rate.

F The Two-Step Procedure: Wealth Effect and Tax Effects

By analytical convenience, we rewrite the system of steady-state equations, assuming that $\delta_K = 0$:

$$\frac{h_k\left[k^N\left(\tilde{p}\right)\right]}{\mu} = r^\star, \tag{93a}$$

$$\frac{1}{\mu}Y^N\left(\tilde{K},\tilde{p},\bar{\lambda},\tau^F,\tau^H\right) - c^N\left(\bar{\lambda},\tilde{p},\tau^c\right) - g^N = 0, \tag{93b}$$

$$r^\star\tilde{b} + Y^T\left(\tilde{K},\tilde{p},\bar{\lambda},\tau^F,\tau^H\right) - c^T\left(\bar{\lambda},\tilde{p},\tau^c\right) - g^T = 0, \tag{93c}$$

together with the intertemporal solvency condition

$$\left(\tilde{b} - b_0\right) = \Phi_1\left(\tilde{K} - K_0\right). \tag{93d}$$

where K_0 and b_0 correspond to the initially predetermined stocks of physical capital and foreign assets, the open economy starting from an initial steady-state at time $\mathcal{T}$. If the fiscal shock is permanent, then $\mathcal{T} = 0$.

Derivation of Steady-State Functions

In a **first step**, we solve the system (93a)-(93c) for $\tilde{p}$, $\tilde{K}$ and $\tilde{b}$ as functions of the marginal utility of wealth, $\bar{\lambda}$, the tax rates on consumption and labor together with the mark-up. Totally differentiating equations (93a)-(93c) yields in matrix form:

$$\begin{pmatrix} h_{kk}k^N_p & 0 & 0 \\ \left(\frac{Y^N_p}{\mu} - c^N_p\right) & \frac{Y^N_K}{\mu} & 0 \\ \left(Y^T_p - c^T_p\right) & Y^T_K & r^\star \end{pmatrix}\begin{pmatrix} \mathrm{d}\tilde{p} \\ \mathrm{d}\tilde{K} \\ \mathrm{d}\tilde{b} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{Y^N_K}{\mu}\mathrm{d}\mu \\ -\left(\frac{Y^N_{\bar{\lambda}}}{\mu} - c^N_{\bar{\lambda}}\right)\mathrm{d}\bar{\lambda} + c^N_{\tau^c}\mathrm{d}\tau^c - \frac{Y^N_{\tau^F}}{\mu}\mathrm{d}\tau^F - \frac{Y^N_{\tau^H}}{\mu}\mathrm{d}\tau^H - \left(\frac{Y^N_\mu}{\mu} - \frac{Y^N}{\mu^2}\right)\mathrm{d}\mu \\ -\left(Y^T_{\bar{\lambda}} - c^T_{\bar{\lambda}}\right)\mathrm{d}\bar{\lambda} + c^T_{\tau^c}\mathrm{d}\tau^c - Y^T_{\tau^F}\mathrm{d}\tau^F - Y^T_{\tau^H}\mathrm{d}\tau^H - Y^T_\mu\mathrm{d}\mu \end{pmatrix}, \tag{94}$$

where we used the fact that $\mu f = p\left[h - h_k\left(k^N - k^T\right)\right]$ and $\frac{h_k}{\mu} = r^\star$ at the steady-state to rewrite $r^\star - h_{kk}k^N_\mu$ as $\frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}^T\right)} = \frac{Y^N_K}{\mu}$.

The equilibrium value of the marginal utility of wealth $\bar{\lambda}$ and tax rates, τ^c, τ^F, τ^H and τ^K

determine the following steady-state values:

$$\tilde{p} = p\left(\tau^K, \mu\right), \tag{95a}$$
$$\tilde{K} = K\left(\bar{\lambda}, \tau^c, \tau^F, \tau^H, \mu\right), \tag{95b}$$
$$\tilde{b} = v\left(\bar{\lambda}, \tau^c, \tau^F, \tau^H, \mu\right), \tag{95c}$$

with partial derivatives given by:

$$K_{\bar{\lambda}} \equiv \frac{\partial \tilde{K}}{\partial \bar{\lambda}} = -\frac{1}{\bar{\lambda}}\frac{1}{\mu_1}\left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L}\tilde{k}^T \mu_1\right) > 0 \quad \text{case} \quad k^T > k^N, \tag{96a}$$
$$= -\frac{1}{\bar{\lambda}}\frac{1}{\mu_2}\left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L}\tilde{k}^T \mu_2\right) > 0 \quad \text{case} \quad k^N > k^T, \tag{96b}$$
$$v_{\bar{\lambda}} \equiv \frac{\partial \tilde{b}}{\partial \bar{\lambda}} = -\frac{1}{\bar{\lambda}}\frac{1}{r^\star \tilde{h}}\left[\sigma_c\left(\tilde{f}\tilde{c}^N + \tilde{h}\tilde{c}^T\right) + \sigma_L \tilde{L}\tilde{h}\tilde{f}\right] < 0, \tag{96c}$$

and

$$K_{\tau^c} \equiv \frac{\partial \tilde{K}}{\partial \tau^c} = -\frac{1}{\mu_1}\left(\frac{\sigma_c \tilde{c}^N}{1+\tau^c}\right) > 0 \quad \text{case} \quad k^T > k^N, \tag{97a}$$
$$= -\frac{1}{\mu_2}\left(\frac{\sigma_c \tilde{c}^N}{1+\tau^c}\right) < 0 \quad \text{case} \quad k^N > k^T, \tag{97b}$$
$$v_{\tau^c} \equiv \frac{\partial \tilde{b}}{\partial \tau^c} = -\frac{1}{r^\star \tilde{h}}\left(\frac{\sigma_c}{1+\tau^c}\right)\left(\tilde{f}\tilde{c}^N + \tilde{h}\tilde{c}^T\right) < 0, \tag{97c}$$

and

$$K_{\tau^F} \equiv \frac{\partial \tilde{K}}{\partial \tau^F} = -\frac{\sigma_L \tilde{L}}{1+\tau^F}\tilde{k}^T < 0, \tag{98a}$$
$$v_{\tau^F} \equiv \frac{\partial \tilde{b}}{\partial \tau^F} = \frac{\tilde{f}}{r^\star}\frac{\sigma_L \tilde{L}}{1+\tau^F} > 0, \tag{98b}$$

and

$$K_{\tau^H} \equiv \frac{\partial \tilde{K}}{\partial \tau^H} = -\frac{\sigma_L \tilde{L}}{1-\tau^H}\tilde{k}^T < 0, \tag{99a}$$
$$v_{\tau^H} \equiv \frac{\partial \tilde{b}}{\partial \tau^H} = \frac{\tilde{f}}{r^\star}\frac{\sigma_L \tilde{L}}{1-\tau^H} > 0. \tag{99b}$$

and

$$
\begin{aligned}
p_\mu \equiv \frac{\partial \tilde{p}}{\partial \mu} &= -\frac{\tilde{p}}{\mu}\frac{\tilde{p}Y_K^N}{\mu Y_K^T} = -\frac{\tilde{p}\mu_1}{\mu\mu_2} > 0, \quad \text{case} \quad k^T > k^N, &(100a)\\
&= -\frac{\tilde{p}\mu_2}{\mu\mu_1} > 0, \quad \text{case} \quad k^N > k^T, &(100b)\\
K_\mu \equiv \frac{\partial \tilde{K}}{\partial \mu} &= \frac{\tilde{p}}{\mu\mu_1\mu_2}\left[\frac{Y_p^N}{\mu} - \mu_1 c_p^N\right] + \frac{Y^N}{\mu^2\mu_1} \lesseqgtr 0, \quad \text{case} \quad k^T > k^N, &(100c)\\
&= \frac{\tilde{p}}{\mu\mu_1\mu_2}\left[\frac{Y_p^N}{\mu} - \mu_2 c_p^N\right] + \frac{Y^N}{\mu^2\mu_2} \lesseqgtr 0, \quad \text{case} \quad k^N > k^T, &(100d)\\
v_\mu \equiv \frac{\partial \tilde{b}}{\partial \mu} &= -\frac{\tilde{p}}{\mu\mu_2}\left[\tilde{p}\left(\frac{Y_p^N}{\mu}\frac{r^\star}{\mu_1} - c_p^N\right) + \left(\sigma_L \tilde{L}\tilde{\Lambda}\tilde{k}^T\mu_1 - \frac{\mu_1}{r^\star}\sigma_c\tilde{c}^N\right)\right] + \frac{\tilde{L}^N\tilde{f}}{\mu r^\star} \gtrless 0,\\
&\qquad \text{case} \quad k^T > k^N &(100e)\\
&= -\frac{\tilde{p}}{\mu\mu_1}\left[\tilde{p}\left(\frac{Y_p^N}{\mu}\frac{r^\star}{\mu_2} - c_p^N\right) + \left(\sigma_L \tilde{L}\tilde{\Lambda}\tilde{k}^T\mu_2 - \frac{\mu_2}{r^\star}\sigma_c\tilde{c}^N\right)\right] + \frac{\tilde{L}^N\tilde{f}}{\mu r^\star} \gtrless 0,\\
&\qquad \text{case} \quad k^N > k^T &(100f)
\end{aligned}
$$

where we used the fact that $h_{kk}k_p^N = -\frac{\mu}{p}\frac{Y_K^T}{p}$ to derive the first equality of (100a). In addition, we made use of the following property $Y_\mu^N = -\frac{p}{\mu}Y_p^N$ and $Y_\mu^T = -\frac{p}{\mu}Y_p^T$ to determine (100c)-(100d) and (100e)-(100f). Finally, use has been made of property (40a) to rewrite $Y_p^T - c_p^T$ and property (40b) to simplify $\mu Y_K^T + \mu Y_K^N$ which is equal to $\tilde{p}\mu r^\star$ in the long-run.

Since the change in the markup modifies the long-run levels of real consumption and labor supply through the steady-state change in the relative price of non tradables, it is convenient to write their steady-state functions by substituting (95a) into their static solutions (27) that hold in the long-run:

$$
c = m\left(\bar{\lambda}, \tau^c, \mu\right), \quad L = n\left(\bar{\lambda}, \tau^F, \tau^H, \mu\right), \tag{101}
$$

where partial derivatives are given by (28) evaluated at the steady-state (that's why we substitute respectively the notations m and n for c and L) and

$$
\begin{aligned}
m_\mu \equiv \frac{\partial \tilde{c}}{\partial \mu} &= \alpha_c\sigma_c\tilde{c}\frac{\mu_1}{\mu_2} < 0, \quad \text{case} \quad k^T > k^N, &(102a)\\
&= \alpha_c\sigma_c\tilde{c}\frac{\mu_2}{\mu_1} < 0, \quad \text{case} \quad k^N > k^T, &(102b)\\
n_\mu \equiv \frac{\partial \tilde{L}}{\partial \mu} &= -\frac{\sigma_L\tilde{L}\tilde{\Lambda}\tilde{k}^T}{\tilde{w}^F}\frac{\tilde{p}\tilde{h}}{\tilde{f}}\frac{\tilde{p}r^\star}{\mu^2} < 0, &(102c)
\end{aligned}
$$

where partial derivatives w. r. t. to $\bar{\lambda}$, τ^c, τ^F, and τ^H are given by (28); we computed (102c) as follows: $n_\mu = \frac{\sigma_L\tilde{L}\tilde{\Lambda}\tilde{k}^T}{\tilde{w}^F}\frac{\tilde{p}Y_K^N}{\mu Y_K^T}\frac{\tilde{p}r^\star}{\mu}$.

Following the same procedure, i. e. substituting the steady-state function for the real exchange rate into the static solution for wage evaluated at the steady-state, the steady-state function for wage writes as follows:

$$
w = w\left(\tau^F, \mu\right), \tag{103}
$$

where the partial derivative w. r. t. μ is given by:

$$w_{\mu} \equiv \frac{\partial \tilde{w}}{\partial \mu} = -\frac{\tilde{k}^T}{1+\tau^F} \frac{\tilde{p}\tilde{h}}{\tilde{f}} \frac{\tilde{p}r^{\star}}{\mu^2} < 0, \tag{104}$$

where $w_{\mu} = \frac{\tilde{k}^T}{1+\tau^F} \frac{\tilde{p}Y_K^N}{\mu Y_K^T} \frac{\tilde{p}r^{\star}}{\mu}$ with $\frac{Y_K^N}{Y_K^T} = -\frac{\tilde{h}}{\tilde{f}} < 0$.

Finally, following a similar procedure, we may express the rental rate of physical capital as a function of τ^F and μ:

$$r^K = r^K\left(\tau^K, \mu\right), \tag{105}$$

where the partial derivative w. r. t. μ is given by:

$$r_{\mu}^K \equiv \frac{\partial \tilde{r}^K}{\partial \mu} = -r^{\star} \frac{\tilde{p}}{\mu} \frac{\mu_1}{\mu_2} > 0, \quad \text{case} \quad k^T > k^N, \tag{106}$$

$$r_{\mu}^K \equiv \frac{\partial \tilde{r}^K}{\partial \mu} = -r^{\star} \frac{\tilde{p}}{\mu} \frac{\mu_2}{\mu_1} > 0, \quad \text{case} \quad k^N > k^T. \tag{107}$$

and the partial derivative w. r. t. τ^F given by (34b).

Derivation of the Equilibrium Value of the Marginal Utility of Wealth

The **second step** consists to determine the equilibrium change of $\bar{\lambda}$ by taking the total differential of the intertemporal solvency condition (93d):

$$\left[v_{\bar{\lambda}} - \Phi_1 K_{\lambda}\right] \mathrm{d}\bar{\lambda} = -\left[v_{\tau^c} - \Phi_1 K_{\tau^c}\right] \mathrm{d}\tau^c - \left[v_{\tau^F} - \Phi_1 K_{\tau^F}\right] \mathrm{d}\tau^F - \left[v_{\tau^H} - \Phi_1 K_{\tau^H}\right] \mathrm{d}\tau^H, \tag{108}$$

from which may solve for the equilibrium value of $\bar{\lambda}$ as a function of tax rates:

$$\bar{\lambda} = \lambda\left(\tau^c, \tau^F, \tau^H\right), \tag{109}$$

with

$$\lambda_{\tau^c} \equiv \frac{\partial \bar{\lambda}}{\partial \tau^c} = -\frac{\left[v_{\tau^c} - \Phi_1 K_{\tau^c}\right]}{\left[v_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right]}, \tag{110a}$$

$$\lambda_{\tau^F} \equiv \frac{\partial \bar{\lambda}}{\partial \tau^F} = -\frac{\left[v_{\tau^F} - \Phi_1 K_{\tau^F}\right]}{\left[v_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right]}, \tag{110b}$$

$$\lambda_{\tau^H} \equiv \frac{\partial \bar{\lambda}}{\partial \tau^H} = -\frac{\left[v_{\tau^H} - \Phi_1 K_{\tau^H}\right]}{\left[v_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right]}. \tag{110c}$$

G Long-Term Effects of a Labor Tax Reform

We first substitute short-run static solutions for consumption, wage and labor given by (27) and (33), into the balanced government budget constraint (8) evaluated at the steady-state:

$$\tau^c p_c(\tilde{p})\, c\left(\bar{\lambda}, \tilde{p}, \tau^c\right) + \left[\left(\tau^F + \tau^H\right) w\left(\tilde{p}, \tau^F\right) - \tau^H \kappa\right] L\left(\bar{\lambda}, \tilde{p}, \tau^F, \tau^H\right) = Z, \tag{111}$$

keeping in mind that the long-run value of the real exchange rate is unaffected by fiscal tax changes and $\bar{\lambda} = \lambda\left(\tau^c, \tau^F, \tau^H\right)$.

G.1 Steady-State Changes of a Tax Reform: Substitution of Payroll Taxes for Consumption Taxes

In this section, we estimate the long-run effects of a fall in the payroll tax τ^F associated with a rise in the consumption tax rate τ^c, which is adjusted accordingly to balance the government budget. Additionally, we assume that taxes on labor income are progressive so that $\kappa > 0$ and $\Lambda < 1$. To avoid confusion, we denote by $\big|^{j,c}$ the effects of the tax reform which involves simultaneously cutting the tax $j = F$ and increasing the tax $k = c$ so that as to leave balanced the government budget condition. In brief, the tax reform strategy involves simultaneously cutting the payroll tax by $\mathrm{d}\tau^F < 0$ and increasing the tax on consumption goods $\mathrm{d}\tau^c|^{F,c} > 0$.

Holding τ^H constant, we differentiate (111)

$$p_c\tilde{c}\mathrm{d}\tau^c\big|^{F,c} + \tau^c p_c \mathrm{d}\tilde{c}\big|^{F,c} + \left[\left(\tau^F + \tau^H\right) w_{\tau^F} + \tilde{w}\right]\mathrm{d}\tau^F + \left(\tilde{w}^F - \tilde{w}^A\right)\mathrm{d}\tilde{L}\big|^{F,c} = 0, \tag{112}$$

with $\left[\left(\tau^F + \tau^H\right) w_{\tau^F} + \tilde{w}\right] = \tilde{w}\left(\frac{1-\tau^H}{1+\tau^F}\right) > 0$.

By using the fact that $\mathrm{d}\tilde{x}\big|^{F,c} = \frac{\mathrm{d}\tilde{x}}{\mathrm{d}\tau^F}\mathrm{d}\tau^F + \frac{\mathrm{d}\tilde{x}}{\mathrm{d}\tau^c}\mathrm{d}\tau^c\big|^{F,c}$, and by rearranging terms, we can determine the size of the rise in the consumption tax rate $\tau^c\big|^{F,c}$ after a fall in the payroll tax τ^F such that the governement budget constraint (111) remains balanced:

$$\mathrm{d}\tau^c\big|^{F,c} = -\frac{\chi_F}{\chi_c}\mathrm{d}\tau^H = -\left\{\frac{\tau^c p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^F} + \left(\tilde{w}^F - \tilde{w}^A\right)\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^F} + \left(\frac{1-\tau^H}{1+\tau^F}\right)\tilde{w}\tilde{L}}{\tau^c p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^c} + \left(\tilde{w}^F - \tilde{w}^A\right)\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^c} + p_c\tilde{c}}\right\}\mathrm{d}\tau^F, \tag{113}$$

where the signs of χ_F and χ_c will be estimated later.

case $k^N > k^T$

If the non traded sector is relatively more capital intensive than the traded sector, the long-run changes are given by:

$$\begin{aligned}
\mathrm{d}\bar{\lambda}\big|^{F,c} &= \lambda_{\tau^F}\mathrm{d}\tau^F + \lambda_{\tau^c}\mathrm{d}\tau^c\big|^{F,c}, \\
&= \frac{\bar{\lambda}}{\chi_c}\frac{p_c\tilde{c}}{\Delta}\left\{\frac{\chi_F}{1+\tau^c}\sigma_c\left(1 + \frac{\sigma_L\tilde{w}^F\tilde{L}}{\sigma_c p_c\tilde{c}}\right) + \sigma_L\tilde{w}^F\tilde{L}\frac{\tilde{\Lambda}}{1+\tau^F}\left(1 - \frac{\tilde{w}^A\tilde{L}}{p_c\left(1+\tau^c\right)\tilde{c}}\right)\right\}\mathrm{d}\tau^F \\
&+ \frac{\bar{\lambda}}{\chi_c\Delta}\frac{\omega_2^1}{\nu_2}\frac{r^\star}{\nu_2}\left(\sigma_c\tilde{c}^N - \sigma_L\tilde{L}k^T\nu_2\tilde{\Lambda}\right)\left[\frac{\chi_F}{1+\tau^c}\left(\sigma_c\tilde{c}^N - \sigma_L\tilde{L}k^T\nu_2\right)\right. \\
&- \left.\sigma_L\tilde{L}k^T\nu_2\frac{\tilde{\Lambda}}{1+\tau^F}p_c\tilde{c}\left(1 - \frac{\tilde{w}^A\tilde{L}}{p_c\left(1+\tau^c\right)\tilde{c}}\right)\right]\mathrm{d}\tau^F < 0, \qquad (114a) \\
\mathrm{d}\tilde{x}\big|^{F,c} &= \frac{\partial\tilde{x}}{\partial\tau^F}\mathrm{d}\tau^F + \frac{\partial\tilde{x}}{\partial\tau^c}\mathrm{d}\tau^c\big|^{F,c} \\
&= \frac{\partial\tilde{x}}{\partial\tau^F}\frac{p_c\tilde{c}}{\chi_c}\left[1 - \frac{\tilde{w}^A\tilde{L}}{p_c\tilde{c}\left(1+\tau^c\right)}\right]\mathrm{d}\tau^F > 0, \qquad (114b)
\end{aligned}$$

where $x = c, K, L, b$. To determine (114b), we used the fact that $\frac{\partial\tilde{x}}{\partial\tau^c} = \frac{\partial\tilde{x}}{\partial\tau^F}\frac{1+\tau^F}{1+\tau^c}\frac{1}{\tilde{\Lambda}}$ and substituted (113), by remembering that $\chi_c = \frac{1+\tau^F}{1+\tau^c}\frac{1}{\tilde{\Lambda}}\left[\chi_F - \frac{\tilde{\Lambda}}{1+\tau^F}\tilde{w}^A\tilde{L}\right] + p_c\tilde{c}$.

We estimated χ_c and χ_F in the case $k^N > k^T$ as follows:

$$
\begin{aligned}
\chi_c &= \tau^c p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^c} + \left(\tilde{w}^F - \tilde{w}^A\right) \frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^c} + p_c \tilde{c} \\
&= \frac{\tilde{w}^F \tilde{L} p_c \tilde{c}}{(1+\tau^c)\Delta} \left\{ \sigma_c (1+\tau^c) \left(\frac{p_c \tilde{c}}{\tilde{w}^F \tilde{L}} - \sigma_L \frac{\tau^c}{1+\tau^c} \right) + \sigma_L \left[(1+\tau^c) - \sigma_c \tau^A \right] \right\} \\
&- (1+\tau^c) \frac{\omega_2^1}{\nu_2} \frac{r^\star}{\nu_2} \left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L} k^T \nu_2 \tilde{\Lambda} \right) \\
&\left\{ \sigma_L \tilde{L} k^T \nu_2 \tilde{\Lambda} \left(1 + \sigma_L \frac{\tau^c}{1+\tau^c} \right) - \sigma_c \tilde{c}^N \left(1 + \sigma_L \frac{\tilde{w}^F \tilde{L}}{p_c (1+\tau^c) \tilde{c}} \tau^A \right) \right\} > 0, \quad (115a) \\
\chi_F &= \tau^c p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^F} + \left(\tilde{w}^F - \tilde{w}^A\right) \frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^F} + \left(\frac{\tilde{\Lambda}}{1+\tau^F} \right) \tilde{w}^A \tilde{L}. \quad (115b)
\end{aligned}
$$

While the sign of χ_c is positive, we are unable to determine the sign of χ_F. However, since it is reasonable to suppose that a rise in the consumption tax rate is required after a fall in the payroll tax rate for the government budget to be balanced, we assume that $\chi_F > 0$.

case $k^T > k^N$

If the traded sector is relatively more capital intensive than the non traded sector, the long-run changes are given by:

$$
\begin{aligned}
\mathrm{d}\tilde{\lambda}\big|^{F,c} &= \lambda_{\tau^F} \mathrm{d}\tau^F + \lambda_{\tau^c} \mathrm{d}\tau^c\big|^{F,c}, \\
&= \frac{\lambda_{\tau^F}}{\chi_c} \left\{ \frac{1+\tau^F}{1+\tau^c} \frac{\chi_F}{\tilde{\Lambda}} \left(1 + \frac{\sigma_c p_c \tilde{c}}{\sigma_L \tilde{w}^F \tilde{L}} \right) + p_c \tilde{c} \left(1 - \frac{\tilde{w}^A \tilde{L}}{p_c \tilde{c} (1+\tau^c)} \right) \right\} \mathrm{d}\tau^F < 0, \quad (116a) \\
\mathrm{d}\tilde{c}\big|^{F,c} &= \frac{\partial \tilde{x}}{\partial \tau^F} \mathrm{d}\tau^F + \frac{\partial \tilde{x}}{\partial \tau^c} \mathrm{d}\tau^c \lambda_{\tau^c} \mathrm{d}\tau^c\big|^{F,c} \\
&= \frac{\partial \tilde{x}}{\partial \tau^F} \frac{p_c \tilde{c}}{\chi_c} \left[1 - \frac{\tilde{w}^A \tilde{L}}{p_c \tilde{c} (1+\tau^c)} \right] \mathrm{d}\tau^F > 0, \quad (116b)
\end{aligned}
$$

where $x = c, K, L, b$. To derive (116a), we made use of the fact that $\lambda_{\tau^c} = -\lambda_{\tau^F} \frac{\sigma_c p_c \tilde{c}}{\sigma_L \tilde{w}^F \tilde{L}} \frac{1}{\tilde{\Lambda}} \frac{1+\tau^F}{1+\tau^c}$. To determine (116b), we used the fact that $\frac{\partial \tilde{x}}{\partial \tau^c} = \frac{\partial \tilde{x}}{\partial \tau^F} \frac{1+\tau^F}{1+\tau^c} \frac{1}{\tilde{\Lambda}}$ and substituted (113), by remembering that $\chi_c = \frac{1+\tau^F}{1+\tau^c} \frac{1}{\tilde{\Lambda}} \left[\chi_F - \frac{\tilde{\Lambda}}{1+\tau^F} \tilde{w}^A \tilde{L} \right] + p_c \tilde{c}$. It is interesting to notice that the fall in the marginal utility of wealth following a rise in τ^c is strengthened by its decrease after a reduction in τ^F.

We estimated χ_c and χ_F in the case $k^T > k^N$ as follows:

$$
\begin{aligned}
\chi_c &= \tau^c p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^c} + \left(\tilde{w}^F - \tilde{w}^A\right) \frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^c} + p_c \tilde{c} \\
&= \frac{\tilde{w}^F \tilde{L} p_c \tilde{c}}{(1+\tau^c)\Delta} \left\{ \sigma_c (1+\tau^c) \left(\frac{p_c \tilde{c}}{\tilde{w}^F \tilde{L}} - \sigma_L \frac{\tau^c}{1+\tau^c} \right) + \sigma_L \left[(1+\tau^c) - \sigma_c \tau^A \right] \right\} > 0, \quad (117a) \\
\chi_F &= \tau^c p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^F} + \left(\tilde{w}^F - \tilde{w}^A\right) \frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^F} + \left(\frac{1-\tau^H}{1+\tau^F} \right) \tilde{w} \tilde{L} \quad (117b) \\
&= \frac{\tilde{w}^F \tilde{L} p_c \tilde{c}}{\Delta} \frac{\tilde{\Lambda}}{1+\tau^F} \left\{ \sigma_c \left(\frac{\tilde{w}^A}{\tilde{w}^F} - \sigma_c \tau^c \right) + \sigma_L \left(\frac{\tilde{w}^A \tilde{L}}{p_c \tilde{c}} - \sigma_c \tau^A \right) \right\} \gtrless 0, \quad (117c)
\end{aligned}
$$

The term χ_c captures two conflictory effects induced by a rise in the consumption tax rate on public revenue. For given levels of real consumption and employment, the increased consumption tax rate raises fiscal earnings as it is reflected by the third term on the RHS of (117a) (first line). However, by increasing the consumption tax rate, households are induced to reduce both their real consumption and their labor supply. Inspection of (117a) (second line) shows that the first term in braces is positive since $\frac{p_c \tilde{c}}{\tilde{w}^F \tilde{L}} > 1$ as longer as $\tilde{a} > 0$ and because $\frac{\tau^c}{1+\tau^c}$ is very small (the average value of the consumption tax rate for 13 OECD countries is approximately 10%). The second term of (117a) (second line) is also positive since σ_c is lower than unity according to empirical evidence and $0 < \tau^A < 1$. Consequently, according to (117a), $\chi_c > 0$; in brief, the decrease in the tax bases via lower consumption and employment is not large enough to more than outweigh the positive tax rate effect.

The sign of χ_F is unclear. Inspection of (117c) shows that the signs of the two terms in braces are ambiguous: [i] because $\frac{\tilde{w}^A}{\tilde{w}^F} < 1$ and $\sigma_L \tau^c < 1$, and since [ii] $\frac{\tilde{w}^A \tilde{L}}{p_c \tilde{c}} < 1$ and $\sigma_c \tau^A < 1$. However, we may reasonably expect that a fall in the payroll tax, i. e. $\mathrm{d}\tau^F < 0$, does not pay for itself so that the rise in the tax bases via higher consumption and higher employment, is not large enough to offset the negative public revenue effect originating from the cut of the labor income tax rate $\mathrm{d}\tau^F$. Consequently, we may assume that $\chi_F > 0$ so that the consumption tax rate must rise for the government balanced budget to hold.

G.2 Steady-State Changes of a Tax Reform: Substitution of labor income Taxes for Consumption Taxes

In this section, we estimate the long-run effects of a fall in the labor income tax τ^H associated with a rise in the consumption tax rate τ^c, which is adjusted accordingly to balance the government budget. Additionally, we assume that taxes on labor income are progressive so that $\kappa > 0$ and $\Lambda < 1$. To avoid confusion, we denote by $\big|^{j,c}$ the effects of the tax reform which involves simultaneously cutting the tax $j = H$ and increasing the tax $k = c$ so that as to to leave balanced the government budget condition. In brief, the tax reform strategy involves simultaneously cutting the labor income tax by $\mathrm{d}\tau^H < 0$ and increasing the tax on consumption goods $\mathrm{d}\tau^c|^{F,c} > 0$.

Differentiate (111) by letting τ^F constant, we can derive the size of the rise in the consumption tax rate:

$$\mathrm{d}\tau^c\big|^{H,c} = -\frac{\chi_H}{\chi_c}\mathrm{d}\tau^H = -\left\{\frac{\tau^c p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^H} + \left(\tilde{w}^F - \tilde{w}^A\right)\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^H} + \left(\tilde{w} - \kappa\right)\tilde{L}}{\tau^c p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^c} + \left(\tilde{w}^F - \tilde{w}^A\right)\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^c} + p_c \tilde{c}}\right\}\mathrm{d}\tau^H. \tag{118}$$

Making use of long-term effects of permanent changes in τ^H and τ^c and substituting $\mathrm{d}\tau^c\big|^{H,c}$ given by (118), we are able to estimate the directions and the sizes of the long-run changes of main economic variables after a fall in τ^H associated with a rise in τ^c by an amount that leaves balanced the government budget constraint. As it is formally shown below, long-run changes are not qualitatively sensitive to sectoral capital-labor ratios. Instead, their magnitude depends on the steady-state variations after a fall in τ^F. Hence, the beneficial effects of a fiscal reform will be sensitive to sectoral capital intensities due to the feed-back effect or "secondary effect" originating from the real exchange rate dynamics over the transition.

case $k^N > k^T$

If the non traded sector is relatively more capital intensive than the traded sector, the long-run changes are given by:

$$\begin{aligned}
\mathrm{d}\bar{\lambda}\big|^{H,c} &= \lambda_{\tau^H}\mathrm{d}\tau^H + \lambda_{\tau^c}\mathrm{d}\tau^c\big|^{H,c}, \\
&= \frac{\bar{\lambda}}{\chi_c}\frac{p_c\tilde{c}}{\Delta}\left\{\frac{\chi_H}{1+\tau^c}\sigma_c\left(1+\frac{\sigma_L\tilde{w}^F\tilde{L}}{\sigma_c p_c\tilde{c}}\right)+\sigma_L\tilde{w}^F\tilde{L}\left(\frac{\tilde{w}-\kappa}{\tilde{w}}\right)\left(1-\frac{\tilde{w}^A\tilde{L}}{p_c\left(1+\tau^c\right)\tilde{c}}\right)\right\}\mathrm{d}\tau^H \\
&+ \frac{\bar{\lambda}}{\chi_c\Delta}\frac{\omega_2^1}{\nu_2}\frac{r^\star}{\nu_2}\left(\sigma_c\tilde{c}^N-\sigma_L\tilde{L}k^T\nu_2\tilde{\Lambda}\right)\left[\frac{\chi_H}{1+\tau^c}\left(\sigma_c\tilde{c}^N-\sigma_L\tilde{L}k^T\nu_2\right)\right. \\
&- \left.\sigma_L\tilde{L}k^T\nu_2\left(\frac{\tilde{w}-\kappa}{\tilde{w}}\right)p_c\tilde{c}\left(1-\frac{\tilde{w}^A\tilde{L}}{p_c\left(1+\tau^c\right)\tilde{c}}\right)\right]\mathrm{d}\tau^H > 0, \qquad (119a) \\
\mathrm{d}\tilde{x}\big|^{H,c} &= \frac{\partial\tilde{x}}{\partial\tau^H}\mathrm{d}\tau^H + \frac{\partial\tilde{x}}{\partial\tau^c}\mathrm{d}\tau^c\big|^{H,c} \\
&= \frac{\partial\tilde{x}}{\partial\tau^H}\frac{p_c\tilde{c}}{\chi_c}\left[1-\frac{\tilde{w}^A\tilde{L}}{p_c\tilde{c}\left(1+\tau^c\right)}\right]\mathrm{d}\tau^H, \qquad (119b)
\end{aligned}$$

where $x = c, K, L, b$. To determine (119b), we used the fact that $\frac{\partial\tilde{x}}{\partial\tau^c} = \frac{\partial\tilde{x}}{\partial\tau^H}\frac{1}{1+\tau^c}\left(\frac{\tilde{w}^A}{\tilde{w}-\kappa}\right)$ and substituted (118), by remembering that $\chi_c = \frac{1}{1+\tau^c}\left(\frac{\tilde{w}^A}{\tilde{w}-\kappa}\right)\left[\chi_H - \left(\tilde{w}-\kappa\right)\tilde{L}\right] + p_c\tilde{c}$.

We estimated χ_c and χ_H in the case $k^N > k^T$ as follows:

$$\begin{aligned}
\chi_c &= \tau^c p_c\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^c} + \left(\tilde{w}^F-\tilde{w}^A\right)\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^c} + p_c\tilde{c} \\
&= \frac{\tilde{w}^F\tilde{L}p_c\tilde{c}}{\left(1+\tau^c\right)\Delta}\left\{\sigma_c\left(1+\tau^c\right)\left(\frac{p_c\tilde{c}}{\tilde{w}^F\tilde{L}}-\sigma_L\frac{\tau^c}{1+\tau^c}\right)+\sigma_L\left[\left(1+\tau^c\right)-\sigma_c\tau^A\right]\right\} \\
&+ \left(1+\tau^c\right)\frac{\omega_2^1}{\nu_2}\frac{r^\star}{\nu_2}\left(\sigma_c\tilde{c}^N-\sigma_L\tilde{L}k^T\nu_2\tilde{\Lambda}\right) \\
&\quad \left\{-\sigma_L\tilde{L}k^T\nu_2\tilde{\Lambda}\left(1+\sigma_L\frac{\tau^c}{1+\tau^c}\right)+\sigma_c\tilde{c}^N\left(1+\sigma_L\frac{\tilde{w}^F\tilde{L}}{p_c\left(1+\tau^c\right)\tilde{c}}\tau^A\right)\right\} > 0, \qquad (120a) \\
\chi_H &= \tau^c p_c\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^H} + \left(\tilde{w}^F-\tilde{w}^A\right)\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^H} + \left(\tilde{w}-\kappa\right)\tilde{L} > 0. \qquad (120b)
\end{aligned}$$

While the sign of χ_c is positive, we are unable to determine the sign of χ_H. However, since it is reasonable to suppose that a rise in the consumption tax rate is required after a fall in the labor income tax rate for the government budget to be balanced, we assume that $\chi_H > 0$.

case $k^T > k^N$

If the traded sector is relatively more capital intensive than the non traded sector, the long-run changes are given by:

$$\begin{aligned}
\mathrm{d}\bar{\lambda}\big|^{H,c} &= \lambda_{\tau^H}\mathrm{d}\tau^H + \lambda_{\tau^c}\mathrm{d}\tau^c\big|^{H,c}, \\
&= \frac{\lambda_{\tau^H}}{\chi_c}\left\{\frac{\chi_H}{1+\tau^c}\left(\frac{\tilde{w}^A}{\tilde{w}-\kappa}\right)\left(1+\frac{\sigma_c p_c \tilde{c}}{\sigma_L \tilde{w}^F \tilde{L}}\right) + p_c\tilde{c}\left(1-\frac{\tilde{w}^A\tilde{L}}{p_c\tilde{c}\left(1+\tau^c\right)}\right)\right\}\mathrm{d}\tau^H < 0, \qquad (121a) \\
\mathrm{d}\tilde{x}\big|^{H,c} &= \frac{\partial\tilde{x}}{\partial\tau^H}\mathrm{d}\tau^H + \frac{\partial\tilde{x}}{\partial\tau^c}\mathrm{d}\tau^c\big|^{H,c} \\
&= \frac{\partial\tilde{x}}{\partial\tau^H}\frac{p_c\tilde{c}}{\chi_c}\left[1-\frac{\tilde{w}^A\tilde{L}}{p_c\tilde{c}\left(1+\tau^c\right)}\right]\mathrm{d}\tau^H, \qquad (121b)
\end{aligned}$$

where $x = c, K, L, b$. To derive (121a), we made use of the fact that $\lambda_{\tau^c} = -\lambda_{\tau^H}\frac{\sigma_c p_c \tilde{c}}{\sigma_L \tilde{w}^F \tilde{L}}\frac{\tilde{w}^A}{(\tilde{w}-\kappa)}$. To determine (121b), we used the fact that $\frac{\partial\tilde{x}}{\partial\tau^c} = \frac{\partial\tilde{x}}{\partial\tau^H}\frac{1}{1+\tau^c}\left(\frac{\tilde{w}^A}{\tilde{w}-\kappa}\right)$ and substituted (118), by remembering that $\chi_c = \frac{1}{1+\tau^c}\left(\frac{\tilde{w}^A}{\tilde{w}-\kappa}\right)\left[\chi_H - \left(\tilde{w}-\kappa\right)\tilde{L}\right] + p_c\tilde{c}$. It is interesting to notice that the fall in the marginal utility of wealth following a rise in τ^c is strengthened by its decrease after a reduction in τ^H.

We estimated χ_c and χ_H in the case $k^T > k^N$ as follows:

$$\begin{aligned}
\chi_c &= \tau^c p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^c} + \left(\tilde{w}^F - \tilde{w}^A\right)\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^c} + p_c\tilde{c} \\
&= \frac{\tilde{w}^F\tilde{L}p_c\tilde{c}}{\left(1+\tau^c\right)\Delta}\left\{\sigma_c\left(1+\tau^c\right)\left(\frac{p_c\tilde{c}}{\tilde{w}^F\tilde{L}} - \sigma_L\frac{\tau^c}{1+\tau^c}\right) + \sigma_L\left[\left(1+\tau^c\right) - \sigma_c\tau^A\right]\right\} > 0, \qquad (122a) \\
\chi_H &= \tau^c p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^H} + \left(\tilde{w}^F - \tilde{w}^A\right)\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^H} + \left(\tilde{w}-\kappa\right)\tilde{L} \\
&= \frac{\tilde{w}^F\tilde{L}p_c\tilde{c}}{\Delta}\left(\frac{\tilde{w}-\kappa}{\tilde{w}^A}\right)\left\{\sigma_c\left(\frac{\tilde{w}^A}{\tilde{w}^F} - \sigma_L\tau^c\right) + \sigma_L\left(\frac{\tilde{w}^A\tilde{L}}{p_c\tilde{c}} - \sigma_c\tau^A\right)\right\} \gtrless 0, \qquad (122b)
\end{aligned}$$

The term χ_c captures two conflictory effects induced by a rise in the consumption tax rate on public revenue. For given levels of real consumption and employment, the increased consumption tax rate raises fiscal earnings as it is reflected by the third term on the RHS of (122a) (first line). However, by increasing the consumption tax rate, households are induced to reduce both their real consumption and their labor supply. Inspection of (122a) (second line) shows that the first term in braces is positive since $\frac{p_c\tilde{c}}{\tilde{w}^F\tilde{L}} > 1$ as longer as $\tilde{a} > 0$ and because $\frac{\tau^c}{1+\tau^c}$ is very small (the average value of the consumption tax rate for 13 OECD countries is approximately 10%). The second term of (122a) (second line) is also positive since σ_c is lower than unity according to empirical evidence and $0 < \tau^A < 1$. Consequently, according to (122a), $\chi_c > 0$; in brief, the decrease in the tax bases via lower consumption and employment is not large enough to more than outweigh the positive tax rate effect.

The sign of χ_H is unclear. Inspection of (122b) shows that the signs of the two terms in braces are ambiguous: [i] because $\frac{\tilde{w}^A}{\tilde{w}^F} < 1$ and $\sigma_L\tau^c < 1$, and since [ii] $\frac{\tilde{w}^A\tilde{L}}{p_c\tilde{c}} < 1$ and $\sigma_c\tau^A < 1$. However, we may reasonably expect that a fall in the labor income tax rate $\mathrm{d}\tau^H < 0$ does not pay itself so that the rise in the tax bases via higher consumption and higher employment, is not large enough to offset the negative public revenue effect originating from the cut of the labor income tax rate $\mathrm{d}\tau^H$. Consequently, we may assume that $\chi_H > 0$ so that the consumption tax rate must rise for the government balanced budget to hold.

H Dynamic Effects of a Tax Reform

This section estimates the dynamic effects of a tax restructuring. Steady-state changes are those derived into the previous section where we estimated the long-run variations such that the rise in τ^c guarantees that the balanced condition for the government holds. In addition, we consider that the change of the tax scheme can be viewed as an unanticipated permanent tax shock, i. e. in the two first tax reforms we considered, the labor and the consumption tax rates are changed simultaneously so as the government budget balanced condition is met and in the third tax reform, the payroll and the labor income tax rates are changes simultaneously so as to leave unchanged the marginal tax wedge.

The stable adjustment of the economy is described by a saddle-path in (K,p)-space. The capital stock, the real exchange rate, and the stock of traded bonds evolve according to:

$$K(t) = \tilde{K} + B_1 e^{\nu_1 t}, \tag{123a}$$

$$p(t) = \tilde{p} + \omega_2^1 B_1 e^{\nu_1 t}, \tag{123b}$$

$$b(t) = \tilde{b} + \Phi_1 B_1 e^{\nu_1 t}, \tag{123c}$$

where $\omega_2^1 = 0$ if $k^T > k^N$ and with

$$B_1 = K_0 - \tilde{K} = -\mathrm{d}\tilde{K}\big|^{j,k},$$

where we made use of the constancy of K at time $t = 0$ (i. e. K_0 is predetermined).

H.1 A Permanent Rise in the Consumption Tax Rate

case $k^N > k^T$

Using the fact that the steady-state value of the real exchange rate remains unaffected by the change of the consumption tax rate, the initial reaction of the relative price of non tradables is given by:

$$\frac{\mathrm{d}p(0)}{\mathrm{d}\tau^c} = -\omega_2^1 \mathrm{d}\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^c} < 0, \tag{124}$$

where $\omega_2^1 < 0$ and the long-run change of the capital stock is given by (85e).

Regarding the initial reaction of real consumption, evaluating first at time $t = 0$ and differentiating the short-run static solution for real consumption (27) w. r. t. τ^c leads to:

$$\begin{aligned}\frac{\mathrm{d}c(0)}{\mathrm{d}\tau^c} &= c_{\bar{\lambda}}\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^c} + c_p\frac{\mathrm{d}p(0)}{\mathrm{d}\tau^c} + c_{\tau^c},\\ &= -\frac{\sigma_c\tilde{c}\sigma_L\tilde{L}}{\Delta(1+\tau^c)}\left\{\tilde{w}^F\left(1+\alpha_c\sigma_c\frac{\tilde{c}^N}{\tilde{p}}\frac{\omega_2^1}{\nu_2}\right) - \tilde{k}^T\frac{\omega_2^1}{\nu_2}\left(\sigma_c\tilde{c}^N\nu_1 - r^\star\sigma_L\tilde{L}\tilde{k}^T\nu_2\right)\right\} \lesseqgtr 0.\end{aligned}$$

Regarding the initial reaction of employment, evaluating first at time $t = 0$ and differentiating the short-run static solution for labor supply (27) w. r. t. τ^c leads to:

$$\frac{\mathrm{d}L(0)}{\mathrm{d}\tau^c} = L_{\bar{\lambda}}\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^c} + L_p\frac{\mathrm{d}p(0)}{\mathrm{d}\tau^c} \lesseqgtr 0, \tag{125}$$

with $L_p = -\sigma_L \frac{\tilde{L}}{\tilde{w}}\frac{\tilde{k}^T}{(1+\tau^F)}\nu_2 < 0$.

Differentiating solutions (123) with respect to time, we are able to compute the directions of trajectories:

$$\dot{K}(t) = I(t) = -\nu_1\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^c}e^{\nu_1 t}\mathrm{d}\tau^c < 0, \tag{126a}$$

$$\dot{p}(t) = -\nu_1\omega_2^1\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^c}e^{\nu_1 t}\mathrm{d}\tau^c = \omega_2^1 I(t) > 0, \tag{126b}$$

$$\dot{b}(t) = -\nu_1\Phi_1\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^c}e^{\nu_1 t}\mathrm{d}\tau^c = \Phi_1 I(t) > 0, \tag{126c}$$

with $\Phi_1 < 0$ and $\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^c} < 0$.

case $k^T > k^N$

With the reversal of capital intensities, both real consumption and total employment fall immediately to their new lower long-run levels. Over the transition, the negative investment flow is exactly matched by a current account surplus, thus leaving unchanged savings.

H.2 A Permanent Fall in the Payroll Tax Rate

case $k^N > k^T$

Regarding the initial reaction of real consumption, evaluating first at time $t = 0$ and differentiating the short-run static solution for real consumption (27) w. r. t. τ^F leads to:

$$\begin{aligned}\frac{\mathrm{d}c(0)}{\mathrm{d}\tau^F} &= c_{\bar{\lambda}}\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^F} + c_p\frac{\mathrm{d}p(0)}{\mathrm{d}\tau^F}, \\ &= -\frac{\sigma_c\tilde{c}\sigma_L\tilde{L}}{\Delta\left(1+\tau^F\right)}\left\{\tilde{w}^F\left(1+\alpha_c\sigma_c\frac{\tilde{c}^N}{\tilde{p}}\frac{\omega_2^1}{\nu_2}\right) - \tilde{k}^T\frac{\omega_2^1}{\nu_2}\left(\sigma_c\tilde{c}^N\nu_1 - r^\star\sigma_L\tilde{L}\tilde{k}^T\nu_2\right)\right\} \lesseqgtr 0\end{aligned} \tag{127}$$

Regarding the initial reaction of employment, evaluating first at time $t = 0$ and differentiating the short-run static solution for labor supply (27) w. r. t. τ^F leads to:

$$\frac{\mathrm{d}L(0)}{\mathrm{d}\tau^F} = L_{\bar{\lambda}}\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^F} + L_p\frac{\mathrm{d}p(0)}{\mathrm{d}\tau^F} + L_{\tau^F} \lesseqgtr 0. \tag{128}$$

case $k^T > k^N$

With the reversal of capital intensities, both real consumption and total employment reach immediately to their new higher long-run levels. Over the transition, the positive investment flow is exactly matched by a current account deficit, thus leaving unchanged savings.

H.3 Dynamic Effects of a Tax Restructuring

First, it is convenient to introduce some notations. We index by the superscript j the impact and steady-state effects induced by a fiscal reform strategy which involves simultaneous cutting the labor tax $\mathrm{d}\tau^j$ (i e. $j = F$ if the payroll tax is reduced or $j = H$ if the labor income tax is lowered) and increasing the tax rate by $\mathrm{d}\tau^k$ ($k = c, H$). While the two first tax reform strategies involves a rise in the consumption tax rate by $\mathrm{d}\tau^c|^{j,c} > 0$ (so as to leave balanced the government budget constraint), the third tax reform strategy involves an increase in the labor income tax rate by $\mathrm{d}tau^H|^{F,H} > 0$ (so as to leave unchanged the marginal tax wedge τ^M).

This leads to long-run changes which can be written as follows:

$$\mathrm{d}\tilde{x}\Big|^{j,k} = \frac{\mathrm{d}\tilde{x}}{\mathrm{d}\tau^j}\Phi^{j,k}\mathrm{d}\tau^j, \quad j = F, H, \quad k = c, H, \tag{129}$$

where $x = c, L, K$ and we denoted by $0 < \Phi^{j,k} < 1$ the scaled-down term of the long-run change after a fall in the labor tax ($j = F, H$). In addition, we denote by Υ^j the positive term:

$$\Upsilon^F = \frac{\tilde{\Lambda}}{1 + \tau^F} > 0, \quad \Upsilon^H = \tilde{w} - \kappa > 0, \tag{130}$$

with $\Upsilon^F = \frac{1}{1+\tau^F}$ and $\Upsilon^H = 1$ if $\kappa = 0$.

case $k^N > k^T$

Using the fact that the steady-state value of the real exchange rate remains unaffected by a tax restructuring, the initial jump of p is formally given by:

$$\mathrm{d}p(0)\Big|^{j,k} = -\omega_2^1 \mathrm{d}\tilde{K}\Big|^{j,k} > 0, \tag{131}$$

where the long-run change of the capital stock is given by (129) (set $x = K$). From the short-run static solution for real consumption (27), the substitution of its long-run change (129) (set $x = c$) together with the initial jump of the real exchange rate (131), the initial reaction of real consumption is given by:

$$\begin{aligned}
\mathrm{d}c(0)\Big|^{j,k} &= \mathrm{d}\tilde{c}\Big|^{j,k} + c_p \mathrm{d}p(0)\Big|^{j,k} = \left[\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^j} - c_p\omega_2^1\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j}\right]\Phi^{j,k}\mathrm{d}\tau^j, \\
&= -\frac{\sigma_c\tilde{c}\sigma_L\tilde{L}}{\Delta}\Upsilon^j\Bigg\{\tilde{w}^F\left(1 + \alpha_c\sigma_c\frac{\tilde{c}^N}{\tilde{p}}\frac{\omega_2^1}{\nu_2}\right) \\
&- r^\star\tilde{k}^T\frac{\omega_2^1}{\nu_2}\left(\sigma_c\tilde{c}^N\nu_1 - \sigma_L\tilde{L}\tilde{k}^T\nu_2\right)\Bigg\}\Phi^{j,k}\mathrm{d}\tau^j \gtrless 0,
\end{aligned} \tag{132}$$

where we used the fact that $c_p = -\sigma_c\alpha_c\frac{\tilde{c}}{\tilde{p}} < 0$, $\Upsilon^j > 0$ (see (130)), $\left(1 + \alpha_c\sigma_c\frac{\tilde{c}^N}{\tilde{p}}\frac{\omega_2^1}{\nu_2}\right) > 0$ (see (69) by setting $\sigma_L = 0$) and $\mathrm{d}\tau^j < 0$ since we considered a fiscal reform strategy which simultaneously involves cutting labor tax and raising the consumption tax rate so as to leave balanced the government budget constraint; in addition, $\Delta > 0$.

By applying a similar procedure to labor, we can derive its reaction once the fiscal reform is implemented, i. e. at time $t = 0$:

$$\begin{aligned}
\mathrm{d}L(0)\Big|^{j,k} &= \mathrm{d}\tilde{L}\Big|^{j,k} + L_p\mathrm{d}p(0)\Big|^{j,k} = \left[\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^j} - L_p\omega_2^1\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j}\right]\Phi^{j,k}\mathrm{d}\tau^j, \\
&= -\frac{\sigma_L\tilde{L}}{\Delta}\Upsilon^j\Bigg\{\sigma_c p_c\tilde{c} + \frac{\omega_2^1}{\nu_2}\Bigg[\sigma_L\tilde{L}\tilde{\Lambda}\tilde{k}^T\nu_2\sigma_c\left(\tilde{c}^N + \frac{p_c\tilde{c}}{\tilde{w}^F\tilde{L}}\tilde{L}\tilde{k}^T\nu_2\right) \\
&+ \frac{r^\star}{\nu_2}\left(\sigma_c\tilde{c}^N - \sigma_L\tilde{L}\tilde{k}^T\nu_2\tilde{\Lambda}\right)\sigma_c\tilde{c}^N\Bigg]\Bigg\}\Phi^{j,k}\mathrm{d}\tau^j \gtrless 0,
\end{aligned} \tag{133}$$

where $\Delta > 0$, $\mathrm{d}\tau^j < 0$, and we used the fact that $L_p = -\frac{\sigma_L \tilde{L} \tilde{\Lambda} \tilde{k}^T \nu_2}{\tilde{w}^F} < 0$, $\Upsilon^j > 0$ (see (130)).

Differentiating solutions (123) with respect to time, we are able to compute the directions of trajectories:

$$\dot{K}(t) = I(t) = -\nu_1 \mathrm{d}\tilde{K}\big|^{j,k} e^{\nu_1 t} > 0, \tag{134a}$$

$$\dot{p}(t) = -\nu_1 \omega_2^1 \mathrm{d}\tilde{K}\big|^{j,k} e^{\nu_1 t} = \omega_2^1 I(t) < 0, \tag{134b}$$

$$\dot{b}(t) = -\nu_1 \Phi_1 \mathrm{d}\tilde{K}\big|^{j,k} e^{\nu_1 t} = \Phi_1 I(t) < 0, \tag{134c}$$

with $\Phi_1 < 0$ and $\mathrm{d}\tilde{K}\big|^{j,k} > 0$.

Regarding consumption and labor supply behavior, their trajectories can be computed by linearizing their short-run static solutions in the neighborhood of the steady-state and differentiating these with respect to time:

$$\dot{c}(t) = c_p \dot{p}(t) > 0, \quad \dot{L}(t) = L_p \dot{p}(t) > 0, \tag{135}$$

with $c_p < 0$ and $L_p < 0$. From impact and steady-state effects, their rising temporal profiles imply that if they rise in the short-run, their initial upward jumps display a smaller size than the size of their long-run changes.

case $k^T > k^N$

Since the real exchange rate remains unaffected by the fiscal shock in the short-run, real consumption and labor jump immediately to their new higher steady-state levels.

Differentiating solutions (123) with respect to time, we are able to compute the directions of trajectories:

$$\dot{K}(t) = I(t) = -\nu_1 \mathrm{d}\tilde{K}\big|^{j,k} e^{\nu_1 t} > 0, \tag{136a}$$

$$\dot{p}(t) = 0, \tag{136b}$$

$$\dot{b}(t) = -\nu_1 \Phi_1 \mathrm{d}\tilde{K}\big|^{j,k} e^{\nu_1 t} = \Phi_1 I(t) < 0. \tag{136c}$$

I Tax Wedge

In line with general practice, payroll taxes are assumed to be proportional and wage income taxes are taken to be progressive. Following Heijdra and Lightart [2009], we define the **average tax wedge** as the difference between the producer wage (paid by the firm) and the purchasing power on consumption goods of after-tax average wage expressed as a percentage of the wage including payroll taxes:

$$\begin{aligned} \tau^A &\equiv \frac{wL\left(1+\tau^F\right) - \left[\left(1-\tau^H\right)w + \tau^H \kappa\right] L}{w^F L}, \\ &\equiv 1 - \frac{\left[\left(1-\tau^H\right) + \frac{\tau^H \kappa}{w}\right]}{\left(1+\tau^F\right)} \end{aligned} \tag{137}$$

where $w^F = w\left(1+\tau^F\right)$. In addition, we denote by τ^M the **marginal tax wedge** as the difference between the producer wage (paid by the firm) and the after-tax marginal wage expressed as a percentage of the producer cost (i. e. including payroll taxes):

$$\begin{aligned}\tau^M &\equiv \frac{wL\left(1+\tau^F\right) - wL\left(1-\tau^H\right)}{w^F L}, \\ &\equiv 1 - \frac{\left(1-\tau^H\right)}{\left(1+\tau^F\right)}.\end{aligned} \tag{138}$$

The closer to unity τ^M, the larger the gap between the wage paid by firms and the real wage received by households.

Using the definition of τ^M given by (138), we can rewrite the average tax wedge as follows:

$$\tau^A \equiv \tau^M - \frac{\tau^H \kappa}{w^F}. \tag{139}$$

Finally, we provide a measure of the degree of tax progressiveness by the means of the coefficient of average tax progression:

$$\Gamma\left(\tau^F, \tau^H, \kappa, p\right) \equiv \tau^M - \tau^A = \frac{\tau^H \kappa}{w^F}, \tag{140}$$

where $w^F = w\left(1+\tau^F\right)$ with $w = w\left(\tau^F, p\right)$.

As the average tax burden τ^A rises with wage rate, the system tax is progressive such that $\Gamma(.) > 0$ which holds as longer as $\kappa > 0$. It is worth emphasizing that our approach which consists to define the average tax together with the marginal tax wedge by taking into account the wage paid by the firm allows for "scaling" the tax burden faced by households in terms of firms' labor cost, the index of average tax progression being expressed in terms of consumption goods; that's why we use the "wedge" label. By abstracting from this "scaling" approach, we would define the marginal and average tax wedges together with the coefficient of average tax progression as follows : $\tau^M \equiv \tau^H w$, $\tau^A \equiv \tau^H (w - \kappa)$ and $\Gamma \equiv \tau^H \kappa > 0$ (as longer as $\kappa > 0$).

J Labor Tax Reform: A Fall in Payroll Taxes and a Rise in Labor Income Taxes

In this section, we consider a labor tax strategy which involves simultaneously cutting a payroll tax by $\mathrm{d}\tau^F < 0$ and increasing the labor income tax by $\mathrm{d}\tau^H > 0$ so as to leave unchanged the marginal tax wedge, i. e. $\mathrm{d}\tau^M = 0$. By making use of (138), the labor tax reform strategy requires a rise in the wage income tax by the following amount:

$$\mathrm{d}\tau^H\big|^{F,H} \equiv -\theta \mathrm{d}\tau^F, \quad \theta \equiv \frac{1-\tau^H}{1+\tau^F} < 1. \tag{141}$$

From (141), the income wage tax must be increased by a smaller amount than the fall in tau^F for leaving unchanged the marginal tax wedge. Because we assumed initial positive payroll taxes so that the denominator of (138) is higher than unity, the fiscal reform keeping the marginal tax wedge constant does not yield to a change in τ^H by the same proportion than the fall in τ^F.

Substituting the static solution for the wage rate (33) that holds in the long-run, and differentiating the coefficient of average tax progression (140) w. r. t. τ^H and τ^F, and the using (141), we find that the fiscal reform raises the degree of average tax progression :

$$\mathrm{d}\Gamma = -\frac{\kappa}{w^F}\theta \mathrm{d}\tau^F > 0, \tag{142}$$

where $\mathrm{d}\tau^F < 0$ since we considered a fall in payroll taxes. The explanation comes from the fact that the wage rate is raised by the same proportion than the fall in τ^F. Consequently, as longer as $\kappa > 0$, the rise in τ^H leads to an increase in Γ.

Making use of long-term effects of permanent changes in τ^F and τ^H and substituting $\mathrm{d}\tau^H\big|^{F,H}$ given by (141), we are able to estimate the directions and the sizes of the long-run changes of main economic variables after a fall in τ^F associated with a rise in τ^H by an amount that leaves unaffected τ^M.

case $k^N > k^T$

If the non traded sector is relatively more capital intensive than the traded sector, the long-run changes are given by:

$$\begin{aligned}
\mathrm{d}\bar{\lambda}\big|^{F,H} &= \lambda_{\tau^H}\mathrm{d}\tau^H\big|^{F,H} + \lambda_{\tau^F}\mathrm{d}\tau^F, \\
&= \frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^F}\frac{\kappa}{\tilde{\Lambda}}\frac{\left(1-\tau^H\right)}{\tilde{w}^A}\mathrm{d}\tau^F = \frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^F}\frac{\kappa}{\tilde{w}}\mathrm{d}\tau^F, && (143a) \\
\mathrm{d}\tilde{x}\big|^{F,H} &= \frac{\partial\tilde{x}}{\partial\tau^H}\mathrm{d}\tau^H\big|^{F,H} + \frac{\partial\tilde{x}}{\partial\tau^F}\mathrm{d}\tau^F \\
&= \frac{\partial\tilde{x}}{\partial\tau^F}\left[1-\theta\frac{\left(\tilde{w}-\kappa\right)\left(1+\tau^F\right)}{\tilde{w}\left(1-\tau^H\right)}\right]\mathrm{d}\tau^F = \frac{\mathrm{d}\tilde{x}}{\mathrm{d}\tau^F}\frac{\kappa}{\tilde{w}}\mathrm{d}\tau^F, && (143b) \\
& && (143c)
\end{aligned}$$

where $x = c, K, L, b$ and we used the fact that $\frac{\partial\tilde{x}}{\partial\tau^H} = \frac{(\tilde{w}-\kappa)\left(1+\tau^F\right)}{\tilde{w}(1-\tau^H)}\frac{\partial\tilde{x}}{\partial\tau^F}$. From (143b), the long-run changes of real consumption, physical capital and employment fiscal tax reform following a substitution of payroll taxes for income wage taxes are proportional to the steady-state variations after a change in τ^F. While the directions of real expenditure in consumption goods and labor supply are ambiguous, we have been able to state that overall capital stock must rise after a fall in wage taxes paid by employers so that $\tilde{K}$ will be permanently increased after the fiscal tax reform. Interestingly, the larger tax allowances κ, the stronger the long-run stimulus of capital accumulation. This comes from the fact that a higher κ softens the fall in the after-tax labor income and therefore the decrease in the marginal benefit of supplying labor and consequently moderates the $t$hat impinges negatively on $\tilde{L}$. In addition, as κ gets larger, the combined effect of a smaller drop of employment and a lower reduction of the after-tax wage income $\tilde{w}^A$ moderates the *wealth effect* and therefore the decline in real expenditure in consumption goods.

It is now convenient to explore the overall effect of the fiscal tax reform on public revenue. First, while the fall in the tax rate paid by employers reduces the government revenues, they are raised by the increased income labor tax rate paid by households. Since the former is levied on a relatively larger tax basis than the former, i. e. τ^H increased by a smaller amount than the fall in τ^F, for given wage rate and employment, labor fiscal revenues decreased. Second, the fiscal reform strategy makes households willing to spend a larger share of their available time to work and to raise their consumption expenditure, public revenues from labor taxes and VAT increase.

To estimate the overall effect of the fiscal reform strategy on public revenues, we differentiate the balanced government budget constraint (111):

$$\mathrm{d}Z = \tau^c p_c \mathrm{d}\tilde{c}\big|^{F,H} + \left(\tilde{w}^F - \tilde{w}^A\right)\mathrm{d}\tilde{L}\big|^{F,H} + \left\{\left[\tilde{w} + \left(\tau^F + \tau^H\right) w_{\tau^F}\right]\mathrm{d}\tau^F + \left(\tilde{w} - \kappa\right)\mathrm{d}\tau^H\big|^{F,H}\right\}\tilde{L}, \tag{144}$$

where the steady-changes in real consumption and labor supply lead to the following variation in public revenues:

$$\begin{aligned} & \tau^c p_c \mathrm{d}\tilde{c}\big|^{F,H} + \left(\tilde{w}^F - \tilde{w}^A\right)\mathrm{d}\tilde{L}\big|^{F,H} \\ = & -\frac{\sigma_L \tilde{L}}{\Delta}\kappa\tilde{\Lambda}\sigma_c\left\{p_c\tilde{c}\left[(1+\tau^c) - \frac{\tilde{w}^A}{\tilde{w}^F}\right] + \frac{r^\star}{\nu_2}\frac{\omega_2^1}{\nu_2}\left(\sigma_c\tilde{c}^N - \sigma_L\tilde{L}\tilde{k}^T\nu_2\right)\left(\tilde{c}^N\tau^A - \frac{p_c\tilde{c}}{\tilde{w}^F}\tau^c\tilde{k}^T\nu_2\right)\right\} \end{aligned} \tag{145}$$

The sign of (145) follows from the fact that consumption and employment rise after a fall in the payroll tax τ^F so that government earnings grow. In addition, we computed the last term on the RHS of (111) which can be rewritten as follows:

$$\left[\tilde{w} + \left(\tau^F + \tau^H\right) w_{\tau^F}\right]\tilde{L}\mathrm{d}\tau^F + \left(\tilde{w} - \kappa\right)\tilde{L}\mathrm{d}\tau^H\big|^{F,H} = \kappa\theta\tilde{L}\tau^F < 0. \tag{146}$$

case $k^T > k^N$

If the traded sector is relatively more capital intensive than the non traded sector, the long-run changes are given by:

$$\begin{aligned} \mathrm{d}\bar{\lambda}\big|^{F,H} &= \lambda_{\tau^H}\mathrm{d}\tau^H\big|^{F,H} + \lambda_{\tau^F}\mathrm{d}\tau^F, \\ &= \frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^F}\frac{\kappa}{\tilde{\Lambda}}\frac{\left(1-\tau^H\right)}{\tilde{w}^A}\mathrm{d}\tau^F = \frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}\tau^F}\frac{\kappa}{\tilde{w}}\mathrm{d}\tau^F, && \text{(147a)} \\ \mathrm{d}\tilde{x}\big|^{F,H} &= \frac{\partial\tilde{x}}{\partial\tau^H}\mathrm{d}\tau^H\big|^{F,H} + \frac{\partial\tilde{x}}{\partial\tau^F}\mathrm{d}\tau^F \\ &= \frac{\partial\tilde{x}}{\partial\tau^F}\left[1 - \theta\frac{\left(\tilde{w}-\kappa\right)\left(1+\tau^F\right)}{\tilde{w}\left(1-\tau^H\right)}\right]\mathrm{d}\tau^F = \frac{\mathrm{d}\tilde{x}}{\mathrm{d}\tau^F}\frac{\kappa}{\tilde{w}}\mathrm{d}\tau^F, && \text{(147b)} \\ & && \text{(147c)} \end{aligned}$$

where $x = c, K, L, b$ and we used the fact that $\frac{\partial\tilde{x}}{\partial\tau^H} = \frac{(\tilde{w}-\kappa)\left(1+\tau^F\right)}{\tilde{w}(1-\tau^H)}\frac{\partial\tilde{x}}{\partial\tau^F}$. From (147b), the long-run changes of real consumption, physical capital and employment fiscal tax reform following a substitution of payroll taxes for income wage taxes are proportional to the steady-state variations after a change in τ^F. Consequently, according to (90), the fiscal tax reform unambiguously raises real consumption and labor supply, and stimulates capital accumulation in the long-run.

By substituting of long-run changes of real consumption and employment, we obtain:

$$\begin{aligned}
\mathrm{d}Z &= \left\{-\frac{\sigma_c p_c \tilde{c} \sigma_L \tilde{L} \tilde{\Lambda}}{\Delta}\left(\tau^c + \tau^A\right) + \kappa \left(\frac{1-\tau^H}{1+\tau^F}\right) \tilde{L}\right\} \mathrm{d}\tau^F, \\
&= \frac{\sigma_L \tilde{w}\left(1-\tau^H\right)\tilde{L}}{\Delta} \tilde{w}\tilde{L}\left\{-\frac{\sigma_c p_c \tilde{c}}{\tilde{w}^A \tilde{L}}\left(\tau^c + \tau^A\right) + \frac{\kappa}{\tilde{w}}\left(\frac{\sigma_c p_c \tilde{c}}{\sigma_L \tilde{w}^F \tilde{L}} + 1\right)\right\} \mathrm{d}\tau^F, \qquad (148)
\end{aligned}$$

The change in government revenues following the fiscal reform strategy is unclear. While the substitution of labor income taxes for payroll taxes improve the earnings of the state by raising real consumption (hence the fiscal basis of consumption taxes) together with labor (hence the fiscal basis of labor taxes), the fall in government revenues due to the insufficient rise in the labor income tax rate plays in opposite direction. The latter effect must predominate over the former effect so that government revenues fall. The explanation is as follows. Since the labor income tax rate must rise by an amount $\frac{\tilde{w}}{\tilde{w}-\kappa}\frac{1-\tau^H}{1+\tau^F}$ to maintain the government budget constraint balanced, the increase in τ^H is not large enough for this condition to be fulfilled, since the labor income tax rate rises by $\frac{1-\tau^H}{1+\tau^F}$ which is smaller.

K Tax Multipliers

In this section, we derive analytical expressions of overall and sectoral tax multipliers.

K.1 Overall Tax Multipliers

Long-Run Tax Multiplier

Because overall output denoted by Y is the sum of traded output Y^T and non traded output measured in terms of the traded good $\frac{p}{\mu}Y^N$, using the fact that $Y^T \equiv Y^T(K, L, p)$ and $Y^N \equiv Y^N(K, L, p)$, remembering that steady-state level of the real exchange rate is unaffected by a tax restructuring, the steady-state change of overall output can be expressed as:

$$\begin{aligned}
\mathrm{d}\tilde{Y}\big|^{j,k} &= \left(Y_K^T + \frac{\tilde{p}}{\mu}Y_K^N\right)\mathrm{d}\tilde{K}\big|^{j,k} + \left(Y_L^T + \frac{\tilde{p}}{\mu}Y_L^N\right)\mathrm{d}\tilde{L}\big|^{j,k}, \\
&= \tilde{p}r^\star \mathrm{d}\tilde{K}\big|^{j,k} + w^F \mathrm{d}\tilde{L}\big|^{j,k} > 0. \qquad (149)
\end{aligned}$$

where we use properties (40b) and (40c) to get (149).

Initial Tax Multiplier

Adopting a similar procedure keeping in mind that the capital stock is initially predetermined, the short-run tax multiplier writes as follows:

$$\begin{aligned}
\mathrm{d}Y(0)\big|^{j,k} &= \left(Y_L^T + \frac{p}{\mu}Y_L^N\right)\mathrm{d}L(0)\big|^{j,k} + \left(\hat{Y}_p^T + \frac{p}{\mu}\hat{Y}_p^N\right)\mathrm{d}p(0)\big|^{j,k}, \\
&= w^F \mathrm{d}L(0)\big|^{j,k} > 0, \qquad (150)
\end{aligned}$$

where we use properties (40c) to get (150); according to property (40a), denoting by a *hat* the partial derivative of Y w. r. t. p for given labor, $\hat{Y}_p^T + \frac{p}{\mu}\hat{Y}_p^N = 0$;

K.2 Sectoral Tax Multipliers

Long-Run Sectoral Tax Multipliers

$k^N > k^T$

We calculate the tax multiplier in the traded sector by differentiating the short-run static solution for Y^T evaluated at the steady-state:

$$\begin{aligned}
\mathrm{d}\tilde{Y}^T\big|^{j,k} &= Y_K^T \mathrm{d}\tilde{K}\big|^{j,k} + Y_L^T \mathrm{d}\tilde{L}\big|^{j,k} = \Phi^{j,k}\left[Y_K^T \frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j} + Y_L^T \frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^j}\right]\mathrm{d}\tau^F, \\
&= \frac{\nu_1}{\nu_2}\frac{\sigma_L \tilde{L}}{\Delta}\sigma_c p_c \tilde{c}\Upsilon^j \Big\{ \left[(1-\alpha_c)\,\tilde{w}^F + r^\star \tilde{p}\tilde{k}^N\right] \\
&+ \alpha_c \tilde{k}^N r^\star \frac{\omega_2^1}{\nu_2}\left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L} k^T \nu_2 \tilde{\Lambda}\right)\Big\}\Phi \mathrm{d}\tau^j > 0, \quad j = F, H, \qquad (151)
\end{aligned}$$

where $\mathrm{d}\tau^j < 0$ since we consider a fall in the labor income tax rate τ^j, $\Phi^{j,c} = \frac{p_c \tilde{c}}{\chi_c}\left[1 - \frac{\tilde{w}^A \tilde{L}}{p_c(1+\tau^c)\tilde{c}}\right] > 0$ (with $j = F, H$) as longer as $\tilde{a} > 0$, and $\Phi^{F,H} = \frac{\tilde{w}}{\kappa} > 0$. To determine (151), we used the fact that $Y_L^T = -\tilde{p}\nu_1 \tilde{k}^N > 0$, $Y_K^T = \tilde{p}\nu_1 < 0$, $\left(\nu_2 \tilde{k}^T + \nu_1 \tilde{k}^N\right) = -\frac{\tilde{w}^F}{\tilde{p}} < 0$ and $\nu_1 + \nu_2 = r^\star$.

We calculate the tax multiplier in the non traded sector by differentiating the short-run static solution for Y^N/μ evaluated at the steady-state:

$$\begin{aligned}
\frac{\tilde{p}}{\mu}\mathrm{d}\tilde{Y}^N\big|^{j,k} &= \frac{\tilde{p}}{\mu} Y_K^N \mathrm{d}\tilde{K}\big|^{j,k} + \frac{\tilde{p}}{\mu} Y_L^N \mathrm{d}\tilde{L}\big|^{j,k} = \Phi^{j,k}\left[Y_K^N \frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j} + \frac{Y_L^N}{\mu}\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^j}\right]\mathrm{d}\tau^j, \\
&= -\frac{\sigma_L \tilde{L}}{\Delta}\sigma_c \tilde{c}^N \Upsilon^j \Big\{\tilde{w}^F - \tilde{k}^T r^\star \frac{\omega_2^1}{\nu_2}\left(\sigma_c \tilde{c}^N - \sigma_L \tilde{L} k^T \nu_2 \tilde{\Lambda}\right)\Big\}\Phi \mathrm{d}\tau^j > 0, \qquad (152)
\end{aligned}$$

where we used the fact that $Y_K^N = \mu\nu_2 > 0$ and $Y_L^N = -\tilde{k}^T \mu\nu_2 < 0$.

$k^T > k^N$

We calculate the tax multiplier in the traded sector by differentiating the short-run static solution for Y^T evaluated at the steady-state:

$$\begin{aligned}
\mathrm{d}\tilde{Y}^T\big|^{j,k} &= Y_K^T \mathrm{d}\tilde{K}\big|^{j,c} + Y_L^T \mathrm{d}\tilde{L}\big|^{j,c} = \Phi^{j,k}\left[Y_K^T \frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j} + Y_L^T \frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^j}\right]\mathrm{d}\tau^F, \\
&= \frac{\nu_2}{\nu_1}\frac{\sigma_L \tilde{L}}{\Delta}\Upsilon^j \sigma_c p_c \tilde{c}\left[(1-\alpha_c)\,\tilde{w}^F + r^\star \tilde{p}\tilde{k}^N\right]\Phi^{j,k}\mathrm{d}\tau^j > 0, \quad j = F, H, \qquad (153)
\end{aligned}$$

where $\Upsilon^j > 0$ and $\Phi^{j,k} > 0$ (as longer as $\tilde{a} > 0$. We used the fact that $Y_L^T = -\tilde{p}\nu_2 \tilde{k}^N < 0$, $Y_K^T = \tilde{p}\nu_2 > 0$, $\left(\nu_1 \tilde{k}^T + \nu_2 \tilde{k}^N\right) = -\frac{\tilde{w}^F}{\tilde{p}} < 0$ and $\nu_1 + \nu_2 = r^\star$ to get (153).

We calculate the tax multiplier in the non traded sector by differentiating the short-run static solution for Y^N/μ evaluated at the steady-state:

$$\begin{aligned}
\frac{\tilde{p}}{\mu}\mathrm{d}\tilde{Y}^N\big|^{j,k} &= \frac{\tilde{p}}{\mu}Y_K^N\mathrm{d}\tilde{K}\big|^{j,k} + \frac{\tilde{p}}{\mu}Y_L^N\mathrm{d}\tilde{L}\big|^{j,k} = \Phi^{j,k}\left[\frac{Y_K^N}{\mu}\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j} + \frac{Y_L^N}{\mu}\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^j}\right]\mathrm{d}\tau^j, \\
&= -\frac{\sigma_L\tilde{L}}{\Delta}\Upsilon^j\sigma_c\tilde{c}^N\tilde{w}^F\Phi^{j,k}\mathrm{d}\tau^j > 0, \quad j = F, H, \qquad (154)
\end{aligned}$$

where we used the fact that $Y_K^N = \mu\nu_1 < 0$ and $Y_L^N = -\tilde{k}^T\mu\nu_1 > 0$ to get (154).

Short-Run Sectoral Tax Multipliers

$k^N > k^T$

Remembering that the short-run solution $Y^T \equiv Y^T(K, L, p)$, using the fact that the capital stock is initially predetermined, $\mathrm{d}L(0)\big|^{j,k} = \mathrm{d}\tilde{L}\big|^{j,k} + L_p\mathrm{d}p(0)\big|^{j,k}$ and $\mathrm{d}p(0)\big|^{j,k} = -\omega_2^1\mathrm{d}\tilde{K}\big|^{j,k}$, the short-run tax multiplier is given by:

$$\begin{aligned}
\mathrm{d}Y^T(0)\big|^{j,k} &= Y_L^T\mathrm{d}L(0)\big|^{j,k} + \hat{Y}_p^T\mathrm{d}p(0)\big|^{j,k}, \\
&= -\Phi^{j,k}\left[\tilde{p}\nu_1\tilde{k}^N\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^j} + Y_p^T\omega_2^1\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j}\right]\mathrm{d}\tau^j \gtrless 0, \qquad (155)
\end{aligned}$$

where we used the fact that $Y_L^T = -\tilde{p}\nu_1\tilde{k}^N > 0$; we denoted by a *hat* the partial derivative of Y^T w. r. t. p for given labor, i. e. $\hat{Y}_p^T < 0$, and we used the fact that $Y_L^T L_p + \hat{Y}_p^T = Y_p^T$. The short-run tax multiplier in the traded sector is the result of two conflictory forces: while the initial stimulus of labor supply induces a labor inflow in the traded sector, the real exchange appreciation shifts away resources from the traded sector towards the non traded sector.

Differentiating the short-run solution for $Y^N \equiv Y^N(K, L, p)$ and remembering that the capital stock is initially predetermined, the short-run tax multiplier is given by:

$$\begin{aligned}
\frac{p}{\mu}\mathrm{d}Y^N(0)\big|^{j,k} &= \frac{p}{\mu}Y_L^N\mathrm{d}L(0)\big|^{j,k} + \frac{p}{\mu}\hat{Y}_p^N\mathrm{d}p(0)\big|^{j,k}, \\
&= -\Phi^{j,k}\left[\tilde{p}\nu_1\tilde{k}^N\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^j} + \frac{Y_p^N}{\mu}\omega_2^1\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j}\right]\mathrm{d}\tau^j \gtrless 0, \qquad (156)
\end{aligned}$$

where we used the fact that $Y_L^N = -\tilde{k}^T\mu\nu_2 < 0$, and we denoted by a *hat* the partial derivative of Y^N w. r. t. p for given labor, i. e. $\hat{Y}_p^N > 0$. The short-run tax multiplier in the non traded sector is the result of two conflictory forces: while the initial stimulus of labor supply induces a labor outflow from the non traded sector, the real exchange appreciation attracts resources in the non traded sector.

$k^T > k^N$

Remembering that the short-run solution $Y^T \equiv Y^T(K, L, p)$, using the fact that the capital stock is initially predetermined and the real exchange is unaffected by a tax restructuring, the short-run tax multiplier is given by:

$$\mathrm{d}Y^T(0)\big|^{j,k} = Y_L^T\mathrm{d}L(0)\big|^{j,k} = Y_L^T\mathrm{d}\tilde{L}\big|^{j,k} < 0, \qquad (157)$$

where we used the fact that $Y_L^T = -\tilde{p}\nu_2\tilde{k}^N < 0$, Differentiating the short-run solution for $Y^N \equiv Y^N(K, L, p)$ and remembering that the capital stock is initially predetermined, the short-run tax multiplier is given by:

$$\frac{p}{\mu}\mathrm{d}Y^N(0)\big|^{j,k} = \frac{p}{\mu}Y_L^N\mathrm{d}L(0)\big|^{j,k} = \frac{p}{\mu}Y_L^N\mathrm{d}\tilde{L}\big|^{j,k} > 0, \tag{158}$$

where we used the fact that $Y_L^N = -\tilde{k}^T\mu\nu_1 > 0$.

L Welfare Analysis

In this section, we investigate the welfare effects of an unanticipated of tax restructuring which involves simultaneously cutting labor tax by $\mathrm{d}\tau^j < 0$ ($j = F, H$) and raising the consumption tax rate or the labor income tax rate by $\mathrm{d}\tau^k > 0$ ($k = c, H$). We denote by ϕ the instantaneous welfare:

$$\phi(t) = u(c(t)) + v(L(t)), \tag{159}$$

and by U its discounted value over an infinite horizon:

$$U = \int_0^\infty \phi(t)\exp(-\delta t)\,\mathrm{d}t. \tag{160}$$

L.1 Instantaneous Welfare

We first linearize the instantaneous utility function (159) in the neighborhood of the steady-state:

$$\phi(t) = \tilde{\phi} + u_c(\tilde{c})(c(t) - \tilde{c}) + v_L(\tilde{L})\left(L(t) - \tilde{L}\right), \tag{161}$$

with $\tilde{\phi}$ given by

$$\tilde{\phi} = u(\tilde{c}) + v\left(\tilde{L}\right). \tag{162}$$

By substituting solutions for $c(t)$ and $L(t)$, we obtain the stable solution for instantaneous welfare:

$$\phi(t) = \tilde{\phi} + [u_c c_p + v_L L_p]\,\omega_2^1\left(K_0 - \tilde{K}\right)e^{\nu_1 t}, \tag{163}$$

where partial derivatives are evaluated at the steady-state, i. e. $u_c = u_c(\tilde{c})$ and $v_L = v_L\left(\tilde{L}\right)$. We estimate the expression in square brackets by making use of the first-order conditions for consumption and labor supply supply decisions evaluated at the steady-state, i. e. $u_c = p_c\bar{\lambda}(1+\tau^c)$ and $v_L = -\bar{\lambda}\tilde{w}^A$. We obtain:

$$u_c c_p + v_L L_p = \bar{\lambda}\left[-\sigma_c\tilde{c}^N(1+\tau^c) + \frac{\tilde{w}^A}{\tilde{w}^F}\sigma_L\tilde{L}\tilde{k}^T\tilde{\Lambda}\frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}^T\right)}\right] \gtrless 0. \tag{164}$$

Evaluate (159) at the steady-state and differentiate, one obtains the long-run change of ϕ after a tax restructuring:

$$\mathrm{d}\tilde{\phi}\big|^{j,k} = \Phi^{j,k}\left[u_c\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^j}\mathrm{d}\tau^j + v_L\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^j}\right]\mathrm{d}\tau^j \gtrless 0, \tag{165}$$

where $0 < \Phi^{j,k} < 1$.

Evaluate (163) at time $t = 0$ and differentiate, we get the initial reaction of ϕ after a tax restructuring:

$$\begin{aligned}
\mathrm{d}\phi(0)\big|^{j,k} &= \mathrm{d}\tilde{\phi}\big|^{j,k} - \left(u_c c_p + v_L L_p\right)\omega_2^1 \mathrm{d}\tilde{K}\big|^{j,k}, \\
&= \Phi^{j,k}\left\{u_c\left[\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^j} - c_p\omega_2^1\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j}\right] + v_L\left[\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^j} - L_p\omega_2^1\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j}\right]\right\}\mathrm{d}\tau^j, \qquad (166)
\end{aligned}$$

where we used (165) to get (166).

Case $k^N > k^T$

If the non traded sector is more capital intensive than the traded sector, the long-run change of ϕ after a tax restructuring writes as follows:

$$\begin{aligned}
\mathrm{d}\tilde{\phi}\big|^{j,k} &= \Phi^{j,k}\bigg\{p_c\bar{\lambda}\left(1+\tau^c\right)\left[1 - \frac{\tilde{w}^A}{\tilde{w}^F}\frac{1}{p_c\left(1+\tau^c\right)}\right]\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^j}\mathrm{d}\tau^j \\
&+ \Gamma^j\bar{\lambda}\frac{\sigma_L\tilde{L}}{\Delta}\tilde{w}^A\frac{\sigma_c p_c\tilde{c}}{\tilde{p}}\frac{r^\star}{\nu_2}\frac{\omega_2^1}{\nu_2}\left(\sigma_c\tilde{c}^N - \sigma_L\tilde{L}\tilde{k}^T\nu_2\tilde{\Lambda}\right)\left(\alpha_c + \frac{\tilde{k}^T\tilde{p}\nu_2}{\tilde{w}^F}\right)\bigg\}\mathrm{d}\tau^j \gtrless 0, \qquad (167)
\end{aligned}$$

where $0 < \Phi^{j,k} < 1$ and $\Gamma^j > 0$. We substituted $u_c = p_c\bar{\lambda}\left(1+\tau^c\right)$ and $v_L = -\bar{\lambda}\tilde{w}^A$, and we used the fact that $\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^j} = \frac{p_c}{\tilde{w}^F}\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^j} - \Gamma^j\frac{\sigma_L\tilde{L}}{\Delta}\frac{r^\star}{\nu_2}\frac{\omega_2^1}{\nu_2}\left(\sigma_c\tilde{c}^N - \sigma_L\tilde{L}\tilde{k}^T\nu_2\tilde{\Lambda}\right)\sigma_c\tilde{c}^N$ to determine (167). Since the sign of the long-run change of instantaneous welfare is not clear-cut, we dot not calculate the initial reaction of ϕ which in particular depends on expression (167).

Case $k^T > k^N$

If the traded sector is more capital intensive than the non traded sector, the long-run change of ϕ after a tax restructuring writes as follows:

$$\mathrm{d}\tilde{\phi}\big|^{j,k} = \Phi^{j,k}p_c\bar{\lambda}\left(1+\tau^c\right)\left[1 - \frac{\tilde{w}^A}{\tilde{w}^F}\frac{1}{1+\tau^c}\right]\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^j}\mathrm{d}\tau^j > 0, \qquad (168)$$

where $0 < \Phi^{j,k} < 1$. We substituted $u_c = p_c\bar{\lambda}\left(1+\tau^c\right)$ and $v_L = -\bar{\lambda}\tilde{w}^A$, and we used the fact that $\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^j} = \frac{p_c}{\tilde{w}^F}\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^j}$ to determine (168).

L.2 Overall Welfare

Until now, we have analyzed the instantaneous welfare implications of an unanticipated permanent fiscal expansion, say at different points of times. To address welfare effects in a convenient way within an intertemporal-maximizing framework, we have to evaluate the discounted value of (159) over the agent's infinite planning horizon. Whereas the change of overall welfare will be estimated numerically, we determine its measure along a transitional path after a tax restructuring.

In order to have a correct and comprehensive measure of welfare, we calculate first the discounted value of instantaneous welfare over the entire planning horizon:

$$\begin{aligned} U &= \frac{\tilde{\phi}}{\delta} + \frac{\left[u_c c_p + v_L L_p\right]\omega_2^1}{r^\star - \nu_1} A_1 \\ &= \frac{\tilde{\phi}}{\delta} + \frac{\phi(0) - \tilde{\phi}}{r^\star - \nu_1}. \end{aligned} \tag{169}$$

The first term on the right hand-side of (169) represents the capitalized value of instantaneous welfare evaluated at the steady-state. The second term on the RHS of (169) vanishes whenever the traded sector is more capital intensive than the non traded sector since the dynamics of the real exchange degenerate. If consumption reacts strongly on impact and labor is not too much responsive, then $\phi(0)$ can overshoot its long-run level which exerts a positive influence on overall welfare.

Case $k^N > k^T$

If the non traded sector is more capital intensive than the traded sector, the long-run change of ϕ after a tax restructuring writes as follows:

$$\begin{aligned} \mathrm{d}U\big|^{j,k} &= \frac{1}{\delta}\mathrm{d}\tilde{\phi}\big|^{j,k} - \frac{\left[u_c c_p + v_L L_p\right]\omega_2^1}{r^\star - \nu_1}\mathrm{d}\tilde{K}\big|^{j,k} \\ &= \frac{1}{r^\star \nu_2}\Phi^{j,k}\left\{u_c\left[\frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^j} - r^\star c_p \omega_2^1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j}\right] + v_L\left[\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^j} - r^\star L_p \omega_2^1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j}\right]\right\}\mathrm{d}\tau^j \gtrless 0, \end{aligned} \tag{170}$$

with

$$\begin{aligned} \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^j} - r^\star c_p \omega_2^1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j} &= \frac{\sigma_L \tilde{L}}{\Delta}\sigma_c \tilde{c}\Upsilon^j\left\{\tilde{p}\nu_1\tilde{k}^N\left[1 + \alpha_c\sigma_c\frac{\tilde{c}^N}{\tilde{p}}\frac{r^\star}{\nu_2}\omega_2^1\right]\right. \\ &+ \left. r^\star \tilde{k}^T \nu_2 \omega_2^1\left[\sigma_c \tilde{c}^N \alpha_c - \sigma_L \tilde{L}\tilde{k}^T\tilde{\Lambda}\right]\right\} < 0, \end{aligned} \tag{171a}$$

$$\begin{aligned} \frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^j} - r^\star L_p \omega_2^1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}\tau^j} &= -\frac{\sigma_L \tilde{L}}{\Delta}\sigma_c \tilde{c}\Upsilon^j\left\{\nu_2\left[1 + \alpha_c\sigma_c\frac{\tilde{c}^N}{\tilde{p}}\frac{r^\star}{\nu_2}\frac{\omega_2^1}{\nu_2}\right]\right. \\ &+ \left. \frac{r^\star}{\nu_2}\frac{\omega_2^1}{\tilde{p}}\sigma_L\tilde{L}\tilde{k}^T\tilde{\Lambda}\left[\alpha_c\left(1-\sigma_c\right) + \frac{\tilde{p}\nu_2\tilde{k}^T}{\tilde{w}^F}\right]\right\} \lesseqgtr 0. \end{aligned} \tag{171b}$$

Case $k^T > k^N$

If the traded sector is more capital intensive than the non traded sector, the long-run change of ϕ after a tax restructuring writes as follows:

$$\mathrm{d}U\big|^{j,k} = \frac{1}{\delta}\mathrm{d}\tilde{\phi}\big|^{j,k} > 0, \tag{172}$$

where $\mathrm{d}\tilde{\phi}\big|^{j,k} > 0$ is given by (168).

M The Case of Endogenous Markup

There are two sectors in the economy: a perfectly competitive sector which produces a traded good denoted by the superscript T and an imperfectly competitive sector which produces a non traded good denoted by the superscript N. We assume that each producer of a unique variety of the non traded good has the following technology $X_j^N = H(\mathcal{K}_j, \mathcal{L}_j)$ with $\mathcal{K}_j$ the capital stock and $\mathcal{L}_j$ labor.

M.1 Framework

The final non traded output, Y^N, is produced in a competitive retail sector using a constant-returns-to-scale production function which aggregates a continuum measure one of sectoral non traded goods:

$$Y^N = \left[\int_0^1 \left(\mathcal{Q}_j^N\right)^{\frac{\omega-1}{\omega}} \mathrm{d}j\right]^{\frac{\omega}{\omega-1}}, \tag{173}$$

where $\omega > 0$ represents the elasticity of substitution between any two different sectoral goods and $\mathcal{Q}_j^N$ stands for intermediate consumption of sector'j variety (with $j \in [0, N]$). The final good producers behave competitively, and the households use the final good for both consumption and investment.

In each of the j sectors, there are $N > 1$ firms producing differentiated goods that are aggregated into a sectoral non traded good by a CES aggregating function. The non traded output sectoral good j writes as:[34]

$$\mathcal{Q}_j^N = N^{-\frac{1}{\epsilon-1}} \left[\int_0^N \left(\mathcal{X}_{i,j}^N\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{174}$$

where $\mathcal{X}_{i,j}^N$ stands for output of firm i in sector j and ϵ is the elasticity of substitution between any two varieties.

Denoting by p and $\mathcal{P}_j$ the relative price of the final good and of the jth variety of the intermediate good, respectively, the profit the final good producer is written as follows:

$$\Pi^N = p\left[\int_0^N \left(\mathcal{Q}_j^N\right)^{\frac{\omega-1}{\omega}} \mathrm{d}j\right]^{\frac{\omega}{\omega-1}} - \int_0^1 \mathcal{P}_j \mathcal{Q}_j^N \mathrm{d}j. \tag{175}$$

Total cost minimizing for a given level of final output gives the (intratemporal) demand function for each input:

$$\mathcal{Q}_j^N = \left(\frac{\mathcal{P}_j}{p}\right)^{-\omega} Y^N, \tag{176}$$

and the price of the final output is given by:

$$p = \left(\int_0^1 \mathcal{P}_j^{1-\omega} \mathrm{d}j\right)^{\frac{1}{1-\omega}}. \tag{177}$$

where $\mathcal{P}_j$ is the price index of sector j and p is the price of the final good.

Within each sector, there is monopolistic competition; each firm that produces one variety $\mathcal{X}_{i,j}^N$ is a price setter. Intermediate output $\mathcal{X}_{i,j}^N$ is produced using capital $\mathcal{K}_{i,j}^N$ and labor $\mathcal{L}_{i,j}^N$:

$$\mathcal{X}_{i,j}^N = H\left(\mathcal{K}_{i,j}^N, \mathcal{L}_{i,j}^N\right). \tag{178}$$

[34] By having the term $N^{-\frac{1}{\epsilon-1}}$ in (174), the analysis abstracts from the variety effect and concentrates solely on the effects of markup variation.

Denoting by $\mathcal{P}_{i,j}$ the price of good i in sector j, the profit function for the jth sector good producer denoted by π_j^N is:

$$\pi_j^N \equiv \mathcal{P}_j N^{-\frac{1}{\epsilon-1}} \left(\int_0^N \left(\mathcal{X}_{i,j}^N \right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^N \mathcal{P}_{i,j} \mathcal{X}_{i,j}^N \mathrm{d}i. \tag{179}$$

The demand faced by each producer $\mathcal{X}_{i,j}^N$ is defined as :

$$\mathcal{X}_{i,j}^N = \left(\frac{\mathcal{P}_{i,j}}{\mathcal{P}_j} \right)^{-\epsilon} \frac{\mathcal{Q}_j^N}{N}, \tag{180}$$

and the price index of sector j is given by:

$$\mathcal{P}_j = N^{-\frac{1}{1-\epsilon}} \left(\int_0^N \mathcal{P}_{i,j}^{1-\epsilon} \mathrm{d}i \right)^{\frac{1}{1-\epsilon}}. \tag{181}$$

Combining (176) and (180), the demand for variety $\mathcal{X}_{i,j}^N$ can be expressed in terms of the relative price of the final non traded good:

$$\mathcal{X}_{i,j}^N = \left(\frac{\mathcal{P}_{i,j}}{\mathcal{P}_j} \right)^{-\epsilon} \left(\frac{\mathcal{P}_j}{p} \right)^{-\omega} \frac{Y^N}{N}. \tag{182}$$

In order to operate, each intermediate good producer must pay a fixed cost denoted by FC measured in terms of the final good which is assumed to be symmetric across firms. Each firm j chooses capital and labor to maximize profits. The profit function for the ith producer in sector j denoted by $\pi_{i,j}^N$ is:

$$\pi_{i,j}^N \equiv \mathcal{P}_j H \left(\mathcal{K}_j^N, \mathcal{L}_j^N \right) - r^K \mathcal{K}_j^N - w^F \mathcal{L}_j^N - pFC. \tag{183}$$

The demands for capital and hours worked are given by the equalities of the markup-adjusted marginal revenues of capital $\frac{\mathcal{P}_j H_K}{\mu}$ and labor $\frac{\mathcal{P}_j H_L}{\mu}$, to the capital rental rate r^K and the producer wage w^F, respectively.

M.2 First-Order Conditions

The current-value Hamiltonian for the j-th firm's optimization problem in the non traded sector writes as follows:

$$\mathcal{H}_j^N = \mathcal{P}_j H \left(\mathcal{K}_j^N, \mathcal{L}_j^N \right) - r^K \mathcal{K}_j^N - w^F \mathcal{L}_j^N - pFC + \eta_j \left[H \left(\mathcal{K}_j^N, \mathcal{L}_j^N \right) - \mathcal{X}_{i,j}^N \right], \tag{184}$$

where $\mathcal{X}_j^N$ stands for the demand for variety j; firm j chooses its price ϱ_j to maximize profits treating the factor prices as given. First-order conditions for the non traded sector write as follows:

$$\begin{aligned} \mathcal{P}_j H_K + \eta H_K &= r^K, \quad , && (185a) \\ \mathcal{P}_j H_L + \eta H_L &= w^F, && (185b) \\ \eta_j &= \mathcal{P}_j' H_j, && (185c) \end{aligned}$$

Combining (185a)-(185b) with (185c), by assuming that firms j are symmetric, yields:

$$\mathcal{P}_j H_K \left(1 - \frac{1}{e_j}\right) = r^K, \tag{186a}$$

$$\mathcal{P}_j H_L \left(1 - \frac{1}{e_j}\right) = w^F, \tag{186b}$$

where we used the fact that $\frac{\mathcal{P}_j'}{\mathcal{P}_j X_{i,j}^N} = -\frac{1}{e_j}$.

We consider a symmetric equilibrium where all firms in the intermediate good sector produce the output level $\mathcal{X}_{i,j}^N = \mathcal{X}^N$ with the same quantities of labor $\mathcal{L}_{i,j}^N = \mathcal{L}^N$ and capital $\mathcal{K}_{i,j}^N = \mathcal{K}^N$. Hence, the aggregate stock of physical capital and hours worked are $K^N = N\mathcal{K}^N$ and $L^N = N\mathcal{L}^N$, respectively. They also set the same price $\mathcal{P}_{i,j} = \mathcal{P}$. Hence, eqs. (177) and (181) imply that $\mathcal{P} = p$.

Defining the markup $\mu = \frac{e}{e-1}$, first-order conditions rewrite as follows:

$$p \frac{H_K}{\mu} = r^K, \tag{187a}$$

$$p \frac{H_L}{\mu} = w^F. \tag{187b}$$

We follow Yang and Heijdra [1993] and Jaimovich and Floetotto [2008] by taking into account the influence of the individual price on the sectoral price index:

$$e(N) = \epsilon - \frac{(\epsilon - \omega)}{N}, \quad N \in (1, \infty). \tag{188}$$

As it will be useful later, we calculate expressions of the partial derivatives of the price-elasticity of demand and the markup with respect to the number of firms:

$$e_N = \frac{\partial e}{\partial N} = \frac{\epsilon - \omega}{N^2} > 0, \quad \mu_N = \frac{\partial \mu}{\partial N} = -\frac{e_N}{(e-1)^2} = -\frac{e_N}{e-1}\frac{\mu}{e} < 0, \tag{189}$$

where we let $\mu = \frac{e}{e-1}$.

We further assume that free entry drives profits down to zero in every non traded sector at each instant of time. Using constant returns to scale in production, i. e. $X = H(K, L) = H_K K + H_L L$, and the zero profit condition, in the aggregate, we have

$$pH\left(K^N, L^N\right) - r^K K^N - w^F L^N - pNFC = 0. \tag{190}$$

Substituting the short-run static solution for non traded output (37), the zero-profit condition (190) can be rewritten as:

$$Y^N\left(K, p, \bar{\lambda}, \tau^F, \tau^H, \mu(N)\right)\left(1 - \frac{1}{\mu(N)}\right) = NFC. \tag{191}$$

M.3 Short-Run Static Solution for the Number of Firms

The zero profit condition can be solved for the number of producers in the non traded sector:

$$N = N\left(K, p, \bar{\lambda}, \tau^F, \tau^H\right), \tag{192}$$

with partial derivatives given by:

$$N_x \equiv \frac{\partial N}{\partial x} = -\frac{Y_x^N \omega_{FC}}{\chi} \gtrless 0, \tag{193}$$

where $x = K, p, \bar{\lambda}, \tau^F, \tau^H$, $\omega_{FC} \equiv NFC/Y^N$ stands for the share of fixed costs in markup adjusted output and we set

$$\chi = \frac{Y^N}{N} \left\{ \left[\eta_{Y^N,\mu} (\mu - 1) + 1\right] \frac{\eta_{\mu,N}}{\mu} - \omega_{FC} \right\}, \tag{194}$$

Inspection of (194) shows that $\chi < 0$ if $\eta_{\mu,N}$ is not too large. This implies that an input inflow ion the non traded sector that raises Y^N and thereby yields to profit opportunities stimulates entry of firms.

M.4 Equilibrium Dynamics and Formal Solutions

Inserting short-run static solutions for non traded output and consumption, given by (37) and (29) respectively, into the non traded good market-clearing condition (9), and inserting short-run static solution for capital-labor ratio in the non traded good sector (31) into the dynamic equation for the real exchange rate (5d), and substituting the short-run static solution for the number of firms (192) yields:

$$\dot{K} = \frac{Y^N \left\{K, p\mu \left[N(K,p)\right]\right\}}{\mu \left[N(K,p)\right]} - c^N(p) - \delta_K K - g^N, \tag{195a}$$

$$\dot{p} = p \left\{ r^\star + \delta_K - \frac{h_k \left(k^N \left\{p, \mu \left[N(K,p)\right]\right\}\right)}{\mu \left[N(K,p)\right]} \right\}. \tag{195b}$$

For clarity purpose, we dropped variables which are constant over time from short-run static solutions.

Linearizing these two equations around the steady-state, and denoting $\tilde{x} = \tilde{K}, \tilde{p}$ the long-term values of $x = K, p$, we obtain in a matrix form:

$$\left(\dot{K}, \dot{p}\right)^T = J \left(K(t) - \tilde{K}, p(t) - \tilde{p}\right)^T, \tag{196}$$

where J is given by

$$J \equiv \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \tag{197}$$

with

$$b_{11} = \frac{Y^N}{\mu} \left[\frac{Y_K^N}{Y^N} - \frac{\mu_N}{\mu} N_K \left(1 - \frac{Y_\mu^N \mu}{Y^N}\right)\right] - \delta_K, \tag{198a}$$

$$b_1 2 = \frac{Y^N}{\mu} \left[\frac{Y_p^N}{Y^N} - \frac{\mu_N}{\mu} N_p \left(1 - \frac{Y_\mu^N \mu}{Y^N}\right)\right] - c_p^N, \tag{198b}$$

$$b_{21} = \frac{p}{\mu} h_{kk} \frac{\mu_N N_K}{\mu} k^N \left(\frac{h_k}{h_{kk} k^N} - \frac{k_\mu^N \mu}{k^N} \right), \tag{198c}$$

$$b_{22} = -\frac{p}{\mu} h_{kk} \left[k_p^N - \frac{\mu_N N_p}{\mu} k^N \left(\frac{h_k}{h_{kk} k^N} - \frac{k_\mu^N \mu}{k^N} \right)\right], \tag{198d}$$

Equilibrium Dynamics

The determinant denoted by Det of the linearized 2×2 matrix (44) is unambiguously negative:

$$\begin{aligned}
\text{Det J} &= b_{11}b_{22} - b_{12}b_{21} \\
&= \left(\frac{Y_K^N}{\mu} - \delta_K\right)\left[\frac{Y_K^T}{\tilde{p}} + \frac{p}{\mu}h_{kk}k^N \frac{\mu_N N_p}{\mu}\left(\frac{h_k}{h_{kk}k^N} - \frac{k_\mu^N \mu}{k^N}\right)\right] \\
&\quad -\frac{\mu_N}{\mu}N_K\left[\frac{Y^N}{\mu}\left(1 - \frac{Y_\mu^N \mu}{Y^N}\right)\frac{Y_K^T}{\tilde{p}} + \left(\frac{Y_p^N}{\mu} - c_p^N\right)\frac{p}{\mu}h_{kk}k^N\left(\frac{h_k}{h_{kk}k^N} - \frac{k_\mu^N \mu}{k^N}\right)\right]
\end{aligned} \quad (199)$$

and the trace denoted by Tr given by

$$\begin{aligned}
\text{Tr J} &= b_{11} + b_{22} = \frac{Y_K^T}{\mu} + \frac{Y_K^N}{p} - \delta_K \\
&\quad -\frac{\mu_N}{\mu}\left[N_K \frac{Y^N}{\mu}\left(1 - \frac{Y_\mu^N \mu}{Y^N}\right) - N_p \frac{p}{\mu}h_{kk}k^N\left(\frac{h_k}{h_{kk}k^N} - \frac{k_\mu^N \mu}{k^N}\right)\right], \\
&= r^\star - \frac{\mu_N}{\mu}N_K \frac{Y^N}{\mu} > 0,
\end{aligned} \quad (200)$$

where we used the fact that $\frac{Y_K^T}{\mu} + \frac{Y_K^N}{p} = \frac{h_k}{\mu} = r^\star + \delta_K$; the positive sign follows from $N_K > 0$ and $\mu_N < 0$.

Characteristic roots from J write as follows:

$$\nu_i \equiv \frac{1}{2}\left\{\text{Tr J} \pm \sqrt{(\text{Tr J})^2 - 4\text{Det J}}\right\} \gtrless 0, \quad i = 1, 2. \quad (201)$$

We denote by $\nu_1 < 0$ and $\nu_2 > 0$ the stable and unstable real eigenvalues, satisfying

$$\nu_1 < 0 < r^\star < \nu_2. \quad (202)$$

Since the system features one state variable, K, and one jump variable, p, the equilibrium yields a unique one-dimensional stable saddle-path.

General solutions are those described by (53) with eigenvector ω_2^i associated with eigenvalue μ_i given by

$$\omega_2^i = \frac{\nu_i - b_{11}}{b_{12}}, \quad (203)$$

Formal Solution for the Stock of Foreign Assets

We first linearize equation (11) around the steady-state:

$$\dot{b}(t) = r^\star\left(b(t) - \tilde{b}\right) + \left[Y_K^T + Y_\mu^T \mu_N N_K\right]\left(K(t) - \tilde{K}\right) + \left[\left(Y_p^T + Y_\mu^T \mu_N N_p\right) - c_p^T\right](p(t) - \tilde{p}). \quad (204)$$

where c_p^T is given by (30b).

Using the fact that $p(t) - \tilde{p} = \omega_2^1\left(K(t) - \tilde{K}\right)$, setting

$$N_1 = \left[Y_K^T + Y_\mu^T \mu_N N_K\right] + \left[\left(Y_p^T + Y_\mu^T \mu_N N_p\right) - c_p^T\right]\omega_2^1, \quad (205)$$

solving for the differential equation and invoking the transversality condition for intertemporal solvency, i. e. equation (12), the stable solution for net foreign assets finally reduces to:

$$b(t) - \tilde{b} = \Phi_1 \left(K(t) - \tilde{K} \right), \tag{206}$$

and the linearized version of the nation's intertemporal budget constraint:

$$\tilde{b} - b_0 = \Phi_1 \left(\tilde{K} - K_0 \right) \tag{207}$$

where we substituted $B_1 \equiv K_0 - \tilde{K}$.

M.5 Stable Solutions for L, N, and w

Linearizing the short-run static solution $N = N\left((K, p\right)$ yields the stable solution for the number of firms:

$$\begin{aligned} N(t) &= \tilde{N} + N_K \left(K(t) - \tilde{K} \right) + N_p \left(p(t) - \tilde{p} \right), \\ &= \tilde{N} + \left(N_K + N_p \omega_2^1 \right) (K(t) - \tilde{K}. \end{aligned} \tag{208}$$

Evaluating at time $t = 0$ and differentiating yields the initial response of the number of firms :

$$\mathrm{d}N(0)|^{j,k} = \mathrm{d}\tilde{N}|^{j,k} - \left\{ N_K + N_p \omega_2^1 \right) \mathrm{d}\tilde{K}|^{j,k}. \tag{209}$$

Linearizing the short-run static solution for labor $L = L\left(p, \mu\right)$, using the fact that $\mu = \mu\left(N\right)$, and substituting the appropriate solutions, the solution for $L(t)$ reads:

$$L(t) = \tilde{L} + L_p \left(p(t) - \tilde{p} \right) + L_\mu \left(\mu(t) - \tilde{\mu} \right), \tag{210}$$

$$= \tilde{L} + L_p \left[\omega_2^1 - \frac{\tilde{p}}{\tilde{\mu}} \mu_N \left(N_K + N_p \omega_2^1 \right) \right] (K(t) - \tilde{K}), \tag{211}$$

where we used the fact that $L_\mu = -\frac{L_p p}{\mu}$. Evaluating at time $t = 0$ and differentiating yields the initial response of employment :

$$\mathrm{d}L(0)|^{j,k} = \mathrm{d}\tilde{L}^{j,k} - L_p \left[\omega_2^1 - \frac{\tilde{p}}{\tilde{\mu}} \mu_N \left(N_K + N_p \omega_2^1 \right) \right] \mathrm{d}\tilde{K}^{j,k}. \tag{212}$$

Linearizing the short-run static solution for the wage rate $w = w\left(p, \mu\right)$ and substituting appropriate solutions yields:

$$\begin{aligned} w(t) &= \tilde{w} + w_p \omega_2^1 \left(K(t) - \tilde{K} \right) + w_\mu \mu_N \left(N(t) - \tilde{N} \right), \\ &= \tilde{w} + w_p \left[\omega_2^1 - \frac{\tilde{p}}{\tilde{\mu}} \mu_N \left(N_K + N_p \omega_1^1 \right) \right] (K(t) - \tilde{K}), \end{aligned} \tag{213}$$

where we used the fact that $w_\mu = -\frac{w_p p}{\mu}$. Evaluating at time $t = 0$ and differentiating yields the initial response of the wage rate:

$$\mathrm{d}w(0)|^{j,k} = \mathrm{d}\tilde{w}^{j,k} - w_p \left[\omega_2^1 - \frac{\tilde{p}}{\tilde{\mu}} \mu_N \left(N_K + N_p \omega_2^1 \right) \right] \mathrm{d}\tilde{K}^{j,k}. \tag{214}$$

M.6 Overall Tax Multipliers

Long-Run Tax Multiplier

Because overall output denoted by Y is the sum of traded output Y^T and non traded output measured in terms of the traded good $\frac{p}{\mu}Y^N$, using the fact that $Y^T \equiv Y^T(K, L, p, \mu)$ and $Y^N \equiv Y^N(K, L, p, \mu)$, remembering that a tax restructuring exerts a long-term effect on the relative price of non tradables, the steady-state change of overall output can be expressed as:

$$\begin{aligned}
\mathrm{d}\tilde{Y}\big|^{j,k} &= \left(Y_K^T + \frac{\tilde{p}}{\mu}Y_K^N\right)\mathrm{d}\tilde{K}\big|^{j,k} + \left(Y_L^T + \frac{\tilde{p}}{\mu}Y_L^N\right)\mathrm{d}\tilde{L}\big|^{j,k}, \\
&= \tilde{p}r^\star \mathrm{d}\tilde{K}\big|^{j,k} + w^F \mathrm{d}\tilde{L}\big|^{j,k} > 0. \qquad (215)
\end{aligned}$$

where we use properties (40b) and (40c) to get (149); according to property (40a), denoting by a *hat* the partial derivative of Y w. r. t. p for given labor, $\hat{Y}_p^T + \frac{p}{\mu}\hat{Y}_p^N = \hat{Y}_\mu^T + \frac{p}{\mu}\hat{Y}_\mu^N = 0$.

Using the fact that $Y^T(t) = c^T(t) + g^T - r^\star b(t) + ca(t) = c^T + g^T + nx(t)$ and $\frac{Y^N(t)}{\mu} = c^N(t) + g^N + I(t)$, the overall output is equal to $Y(t) = p_c\left(p(t)\right)c(t) + g^T + p(t) + g^N + nx(t) + I(t)$. The steady-state change of GDP following a tax reform is equal to:

$$\mathrm{d}\tilde{Y}\big|^{j,k} = \frac{\tilde{Y}^N}{\tilde{\mu}}\mathrm{d}\tilde{p}\big|^{j,k} + p_c\mathrm{d}\tilde{c}\big|^{j,k} + \mathrm{d}\tilde{nx}\big|^{j,k} + \tilde{p}\mathrm{d}\tilde{I}\big|^{j,k}, \qquad (216)$$

where $\mathrm{d}\tilde{nx}\big|^{j,k} = -r^\star \mathrm{d}\tilde{b}\big|^{j,k}$ and $\mathrm{d}\tilde{I}\big|^{j,k} = \delta_K \mathrm{d}\tilde{K}\big|^{j,k}$.

Initial Tax Multiplier

Keeping in mind that the capital stock is initially predetermined, the short-run tax multiplier writes as follows:

$$\begin{aligned}
\mathrm{d}Y(0)\big|^{j,k} &= \left(Y_L^T + \frac{p}{\mu}Y_L^N\right)\mathrm{d}L(0)\big|^{j,k} + \left(\hat{Y}_p^T + \frac{p}{\mu}\hat{Y}_p^N\right)\mathrm{d}p(0)\big|^{j,k} + \left(\hat{Y}_\mu^T + \frac{p}{\mu}\hat{Y}_\mu^N\right)\mu_N \mathrm{d}N(0)\big|^{j,k}, \\
&= w^F \mathrm{d}L(0)\big|^{j,k} > 0, \qquad (217)
\end{aligned}$$

where we use properties (40c) to get (150); according to property (40a), denoting by a *hat* the partial derivative of Y w. r. t. p for given labor, $\hat{Y}_p^T + \frac{p}{\mu}\hat{Y}_p^N = \hat{Y}_\mu^T + \frac{p}{\mu}\hat{Y}_\mu^N = 0$.

Linearizing around the steady-state yields:

$$Y(t) = \tilde{Y} + \frac{\tilde{Y}^N}{\tilde{\mu}}\left(p(t) - \tilde{p}\right) + \left(nx(t) - \tilde{nx}\right) + \tilde{p}\left(I(t) - \tilde{I}\right).$$

Using (216), the initial reaction of GDP is given by:

$$\mathrm{d}Y(0)\big|^{j,k} = \frac{\tilde{Y}^N}{\tilde{\mu}}\mathrm{d}p(0)\big|^{j,k} + p_c\mathrm{d}c(0)\big|^{j,k} + \mathrm{d}nx(0)\big|^{j,k} + \mathrm{d}I(0)\big|^{j,k}, \qquad (218)$$

where $\mathrm{d}c(0)\big|^{j,k} = \mathrm{d}\tilde{c}\big|^{j,k} - c_p\omega_2^1\mathrm{d}\tilde{K}\big|^{j,k}$, $\mathrm{d}p(0)\big|^{j,k} = \mathrm{d}\tilde{p}\big|^{j,k} - \omega_2^1\mathrm{d}\tilde{K}\big|^{j,k}$, $\mathrm{d}I(0) = -\mu_1\mathrm{d}\tilde{K}\big|^{j,k}$ and $\mathrm{d}nx(0)\big|^{j,k} = \mathrm{d}ca(0)\big|^{j,k} = -\mu_1\Phi_1\mathrm{d}\tilde{K}\big|^{j,k}$.

M.7 Sectoral Tax Multipliers

Long-Run Sectoral Tax Multipliers

We calculate the tax multiplier in the traded sector by differentiating the short-run static solution for Y^T evaluated at the steady-state:

$$\mathrm{d}\tilde{Y}^T\big|^{j,k} = Y_K^T \mathrm{d}\tilde{K}\big|^{j,k} + Y_L^T \mathrm{d}\tilde{L}\big|^{j,k} + \hat{Y}_p^T \mathrm{d}\tilde{p}\big|^{j,k} + \hat{Y}_\mu^T \mu_N \mathrm{d}\tilde{N}\big|^{j,k}. \tag{219}$$

where $\hat{Y}_p^T < 0$, $\hat{Y}_\mu^T > 0$ and $\mu_N < 0$.

Using the fact that $Y^T(t) = c^T + g^T + nx(t)$ and totally differentiating yields the steady-state change of traded output following a tax reform:

$$\mathrm{d}\tilde{Y}^T\big|^{j,k} = \mathrm{d}\tilde{c}^T\big|^{j,k} + \mathrm{d}\tilde{nx}\big|^{j,k} \tag{220}$$

where $\mathrm{d}\tilde{nx}\big|^{j,k} = -r^\star \mathrm{d}\tilde{b}\big|^{j,k}$.

We calculate the tax multiplier in the non traded sector by differentiating the short-run static solution for Y^N/μ evaluated at the steady-state:

$$\frac{\tilde{p}}{\mu}\mathrm{d}\tilde{Y}^N\big|^{j,k} = \frac{\tilde{p}}{\mu} Y_K^N \mathrm{d}\tilde{K}\big|^{j,k} + \frac{\tilde{p}}{\mu} Y_L^N \mathrm{d}\tilde{L}\big|^{j,k} + \frac{\tilde{p}}{\mu}\hat{Y}_p^T \mathrm{d}\tilde{p}\big|^{j,k} + \frac{\tilde{p}}{\mu}\hat{Y}_\mu^T \mu_N \mathrm{d}\tilde{N}\big|^{j,k}. \tag{221}$$

where $\hat{Y}_p^N > 0$, $\hat{Y}_\mu^T < 0$ and $\mu_N < 0$.

Using the fact that $\frac{Y^N(t)}{\mu} = c^N(t) + g^N + I(t)$, and totally differentiating yields the steady-state change of non traded output following a tax reform:

$$\frac{1}{\tilde{\mu}}\mathrm{d}\tilde{Y}^N\big|^{j,k} = \mathrm{d}\tilde{c}^N\big|^{j,k} + \mathrm{d}\tilde{I}\big|^{j,k}, \tag{222}$$

where $\mathrm{d}\tilde{I}\big|^{j,k} = \delta_K \mathrm{d}\tilde{K}\big|^{j,k}$.

Short-Run Sectoral Tax Multipliers

$k^N > k^T$

Remembering that the short-run solution $Y^T \equiv Y^T(K, L, p, \mu)$, using the fact that the capital stock is initially predetermined, the short-run tax multiplier is given by:

$$\mathrm{d}Y^T(0)\big|^{j,k} = Y_L^T \mathrm{d}L(0)\big|^{j,k} + \hat{Y}_p^T \mathrm{d}p(0)\big|^{j,k} + \hat{Y}_\mu^T \mu_N \mathrm{d}N(0)\big|^{j,k}, \tag{223}$$

where $\mathrm{d}L(0)\big|^{j,k}$ and $\mathrm{d}N(0)\big|^{j,k}$ are given by (212) and (209), respectively, and $\mathrm{d}p(0)\big|^{j,k} = \mathrm{d}\tilde{p}\big|^{j,k} - \omega_2^1 \mathrm{d}\tilde{K}\big|^{j,k}$.

Linearizing $Y^T(t) = c^T + g^T + nx(t)$ around the steady-state, evaluating at time $t = 0$ and totally differentiating yields the initial of traded output following a tax reform:

$$\mathrm{d}Y^T(0)\big|^{j,k} = \mathrm{d}c^T(0)\big|^{j,k} + \mathrm{d}nx(0)\big|^{j,k} \tag{224}$$

where $\mathrm{d}nx(0)\big|^{j,k} = \mathrm{d}ca(0)\big|^{j,k} = -\mu_1 \Phi_1 \mathrm{d}\tilde{K}\big|^{j,k}$ and $\mathrm{d}c^T(0)\big|^{j,k} = -c_p^T \omega_2^1 \mathrm{d}\tilde{K}\big|^{j,k}$.

Differentiating the short-run solution for $Y^N \equiv Y^N(K, L, p, \mu)$ and remembering that the capital stock is initially predetermined, the short-run tax multiplier is given by:

$$\frac{p}{\mu}\mathrm{d}Y^N(0)\big|^{j,k} = \frac{p}{\mu}Y^N_L \mathrm{d}L(0)\big|^{j,k} + \frac{p}{\mu}\hat{Y}^N_p \mathrm{d}p(0)\big|^{j,k} + \frac{p}{\mu}\hat{Y}^N_\mu \mu_N \mathrm{d}N(0)\big|^{j,k}. \tag{225}$$

where $\mathrm{d}L(0)\big|^{j,k}$ and $\mathrm{d}N(0)\big|^{j,k}$ are given by (212) and (209), respectively, and $\mathrm{d}p(0)\big|^{j,k} = \mathrm{d}\tilde{p}\big|^{j,k} - \omega_2^1 \mathrm{d}\tilde{K}\big|^{j,k}$.

Linearizing $\frac{Y^N(t)}{\mu} = c^N(t) + g^N + I(t)$, around the steady-state, evaluating at time $t = 0$ and totally differentiating yields the initial of non traded output following a tax reform:

$$\frac{1}{\tilde{\mu}}\mathrm{d}\tilde{Y}^N\big|^{j,k} = \mathrm{d}c^N(0)\big|^{j,k} + \mathrm{d}I(0)\big|^{j,k}, \tag{226}$$

where $\mathrm{d}c^{TN}(0)\big|^{j,k} = -c^N_p \omega_2^1 \mathrm{d}\tilde{K}\big|^{j,k}$ and $\mathrm{d}I(0) = -\mu_1 \mathrm{d}\tilde{K}\big|^{j,k}$.

M.8 Substitution of Payroll Taxes for Consumption Taxes: The Case of Endogenous Markup

In this section, we re-estimate the long-run effects of a fall in the payroll tax τ^F associated with a rise in the consumption tax rate τ^c, which is adjusted accordingly to balance the government budget, by allowing the markup to be endogenous.

We first substitute short-run static solutions for consumption, wage rate, and labor, into the balanced government budget constraint (8) evaluated at the steady-state:

$$\tau^c p_c(\tilde{p})\, c\left(\bar{\lambda}, \tilde{p}, \tau^c\right) + \left[\left(\tau^F + \tau^H\right) w\left(\tilde{p}, \tau^F, \tilde{\mu}\right) - \tau^H \kappa\right] L\left(\bar{\lambda}, \tilde{p}, \tau^F, \tau^H, \tilde{\mu}\right) = Z, \tag{227}$$

keeping in mind that the long-run value of the relative is now affected by a change in the tax rate trough the change in the markup.

Holding τ^H constant, we differentiate (227):

$$\begin{aligned} &p_c \tilde{c} \mathrm{d}\tau^c\big|^{F,c} + \tau^c p_c \mathrm{d}\tilde{c}\big|^{F,c} + \left[\tau^c \tilde{c}^N + \left(\tau^F + \tau^H\right) w_p \tilde{L}\right] \mathrm{d}\tilde{p}\big|^{F,c} + \left(\tau^F + \tau^H\right) w_p \tilde{L} \mu_N \mathrm{d}\tilde{N}\big|^{F,c} \\ &+ \left[\left(\tau^F + \tau^H\right) w_{\tau^F} + \tilde{w}\right] \tilde{L} \mathrm{d}\tau^F + \left(\tilde{w}^F - \tilde{w}^A\right) \mathrm{d}\tilde{L}\big|^{F,c} = 0, \end{aligned} \tag{228}$$

with $\left[\left(\tau^F + \tau^H\right) w_{\tau^F} + \tilde{w}\right] = \tilde{w}\left(\frac{1-\tau^H}{1+\tau^F}\right) > 0$.

By using the fact that $\mathrm{d}\tilde{x}\big|^{F,c} = \frac{\mathrm{d}\tilde{x}}{\mathrm{d}\tau^F}\mathrm{d}\tau^F + \frac{\mathrm{d}\tilde{x}}{\mathrm{d}\tau^c}\mathrm{d}\tau^c\big|^{F,c}$, and by rearranging terms, we can determine the size of the rise in the consumption tax rate $\tau^c\big|^{F,c}$ after a fall in the payroll tax τ^F such that the governement budget constraint (227) remains balanced:

$$\mathrm{d}\tau^c\big|^{F,c} = -\frac{\chi_F}{\chi_c}\mathrm{d}\tau^F, \tag{229}$$

where χ_F and χ_c are given by:

$$\begin{aligned}
\chi_F &= \tau^c p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^F} + \left(\tilde{w}^F - \tilde{w}^A\right) \frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^F} + \left(\frac{1-\tau^H}{1+\tau^F}\right) \tilde{w}\tilde{L} \\
&+ \left[\tau^c \tilde{c}^N + \left(\tau^F + \tau^H\right) w_p \tilde{L}\right] \frac{\mathrm{d}\tilde{p}}{\mathrm{d}\tau^F} + \left(\tau^F + \tau^H\right) w_p \tilde{L} \mu_N \frac{\mathrm{d}\tilde{N}}{\mathrm{d}\tau^F} > 0, \quad (230a) \\
\chi_c &= \tau^c p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}\tau^c} + \left(\tilde{w}^F - \tilde{w}^A\right) \frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau^c} + p_c \tilde{c} \\
&+ \left[\tau^c \tilde{c}^N + \left(\tau^F + \tau^H\right) w_p \tilde{L}\right] \frac{\mathrm{d}\tilde{p}}{\mathrm{d}\tau^c} + \left(\tau^F + \tau^H\right) w_p \tilde{L} \mu_N \frac{\mathrm{d}\tilde{N}}{\mathrm{d}\tau^c} > 0. \quad (230b)
\end{aligned}$$

The size of the rise in τ^c will be estimated numerically.

N Data and Estimation Methodology

The calibration of output shares of capital income (θ^T, θ^N) and markup (μ^N) are based on the EU KLEMS sectoral database, comprising 11 industries and 13 countries over the period 1970-2004.[35] Following De Gregorio, Giovannini and Wolf [1994], Agriculture, Hunting, Forestry and Fishing; Mining and Quarrying; Total Manufacturing; Transport and Storage; and Communication are classified as traded goods with weights given by relative nominal value added within the sector. Electricity, Gas and Water Supply; Construction; Wholesale and Retail Trade; Hotels and Restaurants; Finance, Insurance, Real Estate and Business Services; and Community Social and Personal Services industries account for the non traded sector. The dataset enables construction of aggregate data on traded and non traded output, capital stock and employment. Sectoral ratios Y^N/Y and L^N/L immediately follow.

Markups are estimated at the industry level for each country and are aggregated as follows to construct μ^T and μ^N:

$$\mu^T = \sum_{j=1}^{4} \omega_{jT}\, \hat{\mu}_j \qquad \mu^N = \sum_{j=1}^{6} \omega_{jN}\, \hat{\mu}_j, \tag{231}$$

where ω_{jT} (resp. ω_{jN}) is the nominal value added-weight of industry j in sector T (resp. N). The estimates $\hat{\mu}_j$ are obtained applying the consistent methodology developed by Roeger [1995]. This approach takes account of labor, capital and intermediate inputs as production factors and the specific variables required to apply the Roeger's method are the following: gross output (at basic current prices), compensation of employees, intermediate inputs at current purchasers prices, and capital services (volume) indices. The testable equation of the Roeger's methodology may be written as:

$$y_{j,t} = \beta_j\, x_{j,t} + \varepsilon_{j,t}, \tag{232}$$

with $y_j, t = \Delta\left(p_{j,t}Y_{j,t}\right) - \alpha_{N,t}\Delta\left(w_{j,t}L_{j,t}\right) - \alpha_{M,t}\Delta\left(m_{j,t}M_{j,t}\right) - \left(1-\alpha_{N,t}-\alpha_{M,t}\right)\Delta\left(r_t K_{j,t}\right)$, $x_{j,t} = \Delta\left(p_{j,t}Y_{j,t}\right) - \Delta\left(r_t K_{j,t}\right)$, and $\varepsilon_{j,t}$ the i.i.d. error term. $\Delta\left(p_{j,t}Y_{j,t}\right)$ denotes the nomi-

[35] Austria, Belgium, Denmark, Spain, Finland, France, Germany, Italy, Japan, Netherlands, Sweden, the United Kingdom (UK) and the United States (US).

nal output growth in industry j, $\Delta\left(w_{j,t}L_{j,t}\right)$ the nominal labor cost growth, $\Delta\left(m_{j,t}M_{j,t}\right)$ the growth in nominal intermediate input costs and $\Delta\left(r_tK_{j,t}\right)$ the nominal capital cost growth. All these variables are compiled from the EU KLEMS database except the user cost of capital r_t. No sector-specific information was available to construct r_t, so the rental price of capital is calculated as $r_t(\equiv r_{j,t}) = p_I\left(i - \pi_{GDP} + \delta_K\right)$, with p_I is the deflator for business non residential investment, i the long-term nominal interest rate, π_{GDP} the GDP deflator based inflation rate and the depreciation rate is fixed at 5% throughout (p_I, i and π_{GDP} were taken from OECD database). An econometric issue faced when estimating (232) with the OLS is the potential endogeneity of the regressor associated with the heteroskedasticity and autocorrelation of the error term. To tackle these problems, we estimate (232) by using heteroskedastic and autocorrelation consistent standard errors as suggested by Newey and West [1993] (lag truncation =2). Finally, the markup estimate $\hat{\mu}_j$ is equal to $1/(1-\hat{\beta}_j)$.[36]

Sectoral government expenditure data over the period 1978-2004 were obtained from the Government Finance Statistics Yearbook and OECD database. Following Morshed and Turnovsky [2004], the following four sectors were treated as traded: Fuel and Energy; Agriculture, Forestry, Fishing, and Hunting; Mining, Manufacturing, and Construction; Transport and Communications. The following sectors were treated as being non traded: Government Public Services; Defense; Public Order and Safety; Education; Health; Social Security and Welfare; Housing and Community Amenities; Recreation Cultural and Community Affairs.

Disaggregation of gross fixed capital formation in OECD countries distinguishes between five types of investment inputs. Products of Agriculture, Forestry, Fishing and Aquaculture; Metal Products and machinery; and Transport Equipment are defined as traded inputs. Housing; and Other Buildings are treated as non traded investment goods (source: OECD Input-Output database).

Payroll tax rate, labor income tax rate, and the consumption tax rate are specified as effective tax rates and are computed according to the following formulas:

$$\tau^F = \frac{\text{Taxes on payroll and workforce} + \text{Employers' contribution to social security}}{\text{Compensation of employees}},$$

$$\tau^H = \text{Income tax (average rate)} + \text{Employees' social security contributions (average rate)},$$

$$\tau^c = \frac{\text{Taxes on production, sale, transfer}}{\text{Final consumption expenditure of households and general government}}.$$

Tax allowances, κ, is calculated as the share of taxable income into the gross wage earnings before taxes. All taxes and incomes data were taken from OECD database.

Finally, data for private consumption, government and investment expenditure, as well

[36] Countries estimates for each $\hat{\mu}_j$, $j = 1, ..., 11$, are not reported here to save space, but are available upon request.

those for GDP were drawn from the OECD "National Accounts of OECD Countries" database.

Table (4) summarizes the values of the non tradable share in overall GDP, total employment, public spending, and sectoral output shares of capital income, and, markups.

Table 4: Ratios for Countries

	$\frac{Y^N}{Y}$	$\frac{L^N}{L}$	$\frac{I^N}{I}$	$\frac{g^N}{Y^N}$	$\frac{g^T}{Y^T}$	$\frac{c}{Y}$	$\frac{g}{Y}$	$\frac{I}{Y}$	θ^T	θ^N	μ
Austria	0.65	0.60	0.59	0.28	0.07	0.55	0.20	0.24	0.28	0.32	1.52
Belgium	0.67	0.65	n.a.	0.30	0.09	0.54	0.24	0.22	0.33	0.35	1.39
Denmark	0.70	0.67	0.58	0.40	0.07	0.52	0.29	0.21	0.32	0.32	1.52
Spain	0.61	0.59	0.64	0.25	0.05	0.61	0.16	0.24	0.41	0.33	1.37
Finland	0.58	0.57	0.63	0.34	0.09	0.51	0.23	0.25	0.35	0.26	1.41
France	0.69	0.64	0.63	0.33	0.06	0.56	0.24	0.21	0.27	0.30	1.42
Germany	0.64	0.61	0.61	0.30	0.06	0.57	0.22	0.23	0.22	0.35	1.55
Italy	0.63	0.56	0.54	0.29	0.06	0.58	0.20	0.23	0.22	0.33	1.73
Japan	0.64	0.61	0.59	n.a.	n.a.	0.53	0.14	0.30	0.42	0.39	1.63
Netherlands	0.67	0.69	0.63	0.34	0.08	0.50	0.25	0.22	0.37	0.29	1.36
Sweden	0.65	0.67	0.47	0.43	0.09	0.49	0.31	0.20	0.30	0.30	1.44
UK	0.62	0.66	0.52	0.33	0.05	0.60	0.23	0.19	0.30	0.28	1.47
US	0.68	0.72	0.59	0.22	0.06	0.66	0.17	0.19	0.36	0.32	1.42
Total average	0.65	0.63	0.59	0.32	0.07	0.56	0.22	0.23	0.32	0.32	1.48
$k^T > k^N$	0.64	0.64	0.60	0.30	0.07	0.57	0.20	0.23	0.37	0.31	1.44
$k^N > k^T$	0.66	0.63	0.57	0.33	0.07	0.55	0.24	0.22	0.28	0.33	1.51

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